Matrix algebra

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1 Determinant of a block matrix

We seek a formula for the determinant of a block matrix

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix}$$

Suppose $\mathbf{D}$ is invertible. Factor:

$$\begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{I} & \mathbf{0} \\ -\mathbf{D}^{-1}\mathbf{C} & \mathbf{I} \end{pmatrix} = \begin{pmatrix} \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C} & \mathbf{B} \\ \mathbf{0} & \mathbf{D} \end{pmatrix},$$

so

$$\det \begin{pmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{pmatrix} = \det(\mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}) \cdot \det(\mathbf{D}) = \det(\mathbf{A}\mathbf{D} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}\mathbf{D})$$

So if crossing out all rows and corresponding columns except n of them gives a matrix that is easy to invert, then you can simplify the evalution of the determinant.

References

[1] John R. Silvester, Determinants of Block Matrices, `http://www.mth.kcl.ac.uk/~jrs/gazette/blocks.pdf`

[2] `http://en.wikipedia.org/wiki/Determinant`