Magnetic reconnection for 10-moment two-fluid versus kinetic simulations

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Abstract

We compare two-fluid simulations of the GEM challenge problem with published simulations using kinetic models (Vlasov and PIC). The ten-moment model with pressure isotropization agrees well with kinetic models in the rate of reconnection, the qualitative structure of the electron diffusion region, and the dominance of the pressure term in Ohm's law and is computationally inexpensive due to the absence of diffusive terms.

GEM challenge problem

The GEM magnetic reconnection challenge problem[a] was formulated to compare the ability of different plasma models to model fast magnetic reconnection. It initiates reconnection by pinching adjacent oppositely directed field lines from their equilibrium state.

[a] Birn et al, *Geospace environmental modeling (GEM) magnetic reconnection challenge*, Journal of Geophysical Research—Space Physics, 106:3715–3719, 2001.

Ten-moment two-fluid model

We neglected all interspecies collision terms. The ten-moment two-fluid model we used assumes (1) conservation of mass and momentum and pressure tensor evolution for each species s:

$$\partial_t \rho_s + \nabla\cdot(\rho_s \mathbf{u}_s) = 0,$$

$$\partial_t(\rho_s \mathbf{u}_s) + \nabla\cdot(\rho_s \mathbf{u}_s\mathbf{u}_s + \mathbb{P}_s) = \frac{q_s}{m_s}\rho_s(\mathbf{E} + \mathbf{u}_s\times\mathbf{B}),$$

$$\partial_t \mathbb{P}_s + \nabla\cdot(\mathbf{u}_s\mathbb{P}_s) + 2\,\mathrm{Sym}\,(\mathbb{P}_s\cdot\nabla\mathbf{u}_s) + \nabla\cdot\mathbb{q}_s = 2\,\mathrm{Sym}\,(\frac{q_s}{m_s}\mathbb{P}_s\times\mathbf{B}) + \mathbb{R}_s,$$

and (2) Maxwell's equations for evolution of electromagnetic field:

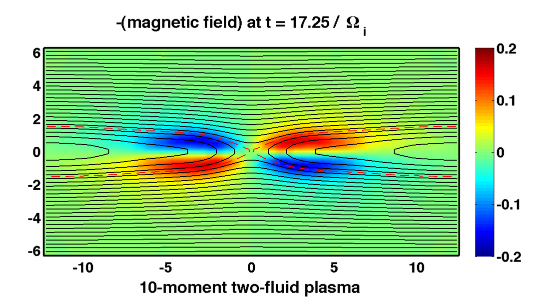
$$\partial_t \mathbf{B} + \nabla\times\mathbf{E} = 0, \qquad \nabla\cdot\mathbf{B} = 0,$$

$$\partial_t \mathbf{E} - c^2\nabla\times\mathbf{B} = -\mathbf{J}/\epsilon, \qquad \nabla\cdot\mathbf{E} = \sigma/\epsilon.$$

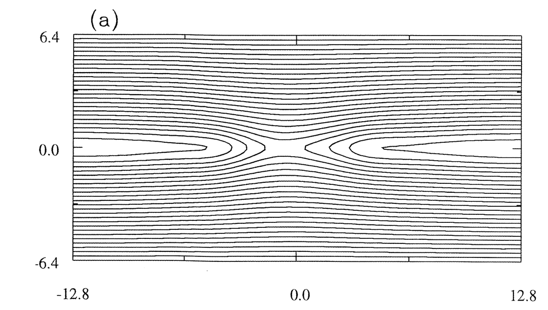
To provide for isotropization we let $\mathbb{R}_s = \frac{1}{\tau_s}\left(\frac{1}{3}(\mathrm{tr}\,\mathbb{P}_s)\mathbb{I} - \mathbb{P}_s\right)$, and for the isotropization period we used $\tau_s = \tau_0\sqrt{\frac{\det \mathbb{P}_s}{\rho_s^5}}m_s^3$, which attempts to generalize the Braginskii closure; for the GEM problem this means that $\tau_i/\tau_e \approx (m_i/m_e)^{5/4}$. Isotropization provides hyperbolic viscosity; rapid isotropization is asymptotically equivalent to a small viscosity $\eta \approx p_s\tau_s$. We set $\tau_0 = 50$. We set $\mathbb{q}_s = 0$. For conservation and shock-capturing purposes we evolve the *energy* tensor $\mathbb{E}_s := \mathbb{P}_s + \rho_s\mathbf{u}_s\mathbf{u}_s$ rather than the pressure tensor. We implemented an explicit third-order discontinuous Galerkin solver in the DoGPack framework.

Magnetic field ($\mathbb{B}$)

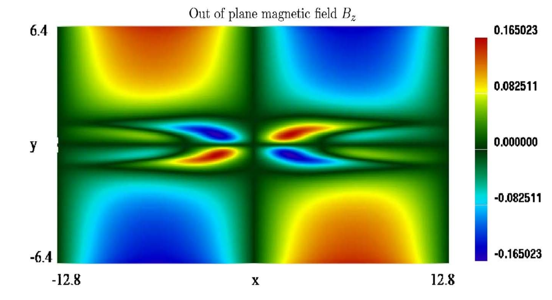
The shape of the magnetic field for our 10-moment simulations agrees well with kinetic (and 5-moment) simulations.



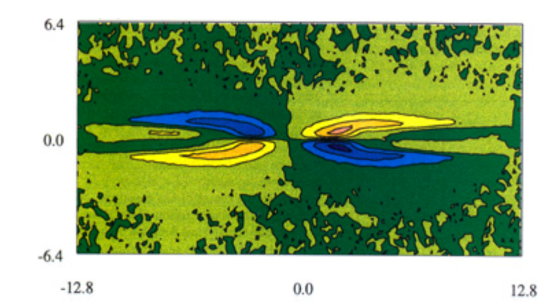
10-moment two-fluid B at $t = 17.25$ (ours)



PIC B lines at $t = 15.7$, quoted from **[Pritchett01]**.



Vlasov B_z at $t = 17.7$, quoted from **[ScGr06]**

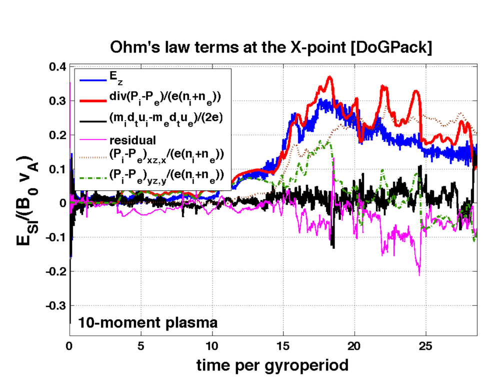


PIC B_z at $t = 15.7$, quoted from **[Pritchett01]**

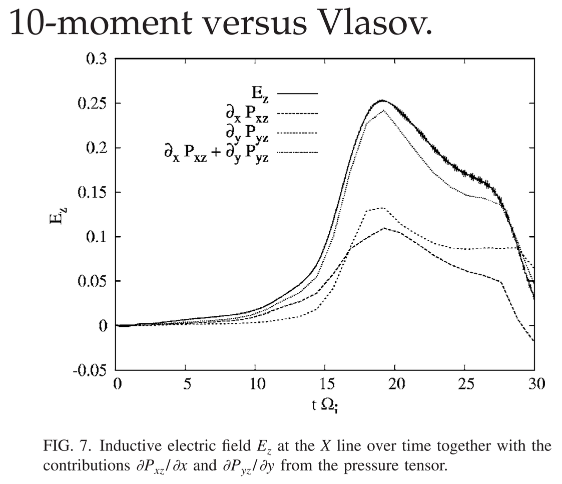
Reconnection Rates

Unlike the hyperbolic 5-moment model, the ten-moment model gets Ohm's law right: in agreement with theoretical arguments and kinetic simulations, the reconnection electric field is primarily supported by nongyrotropic pressure.

Terms of Ohm's law at the X-point for 10-moment versus Vlasov.

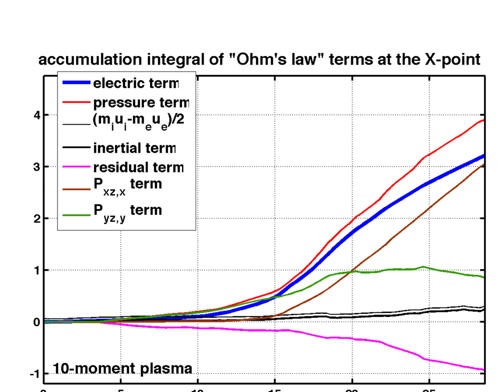


10-moment Ohm's law terms

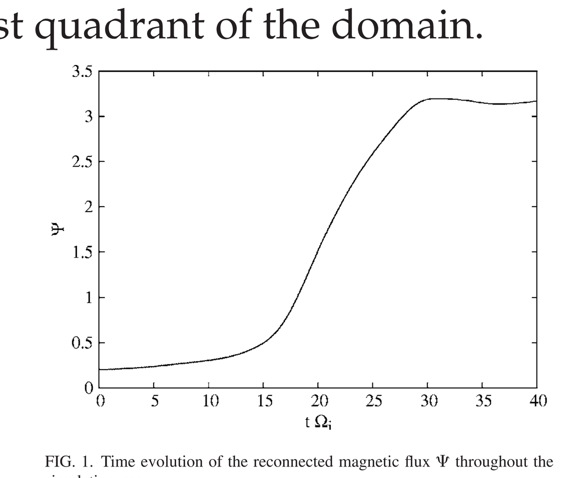


Vlasov, quoted from **[ScGr06]**

Reconnecting flux for 10-moment versus Vlasov. Fluxes are through the boundaries of the first quadrant of the domain.



10-moment reconnection



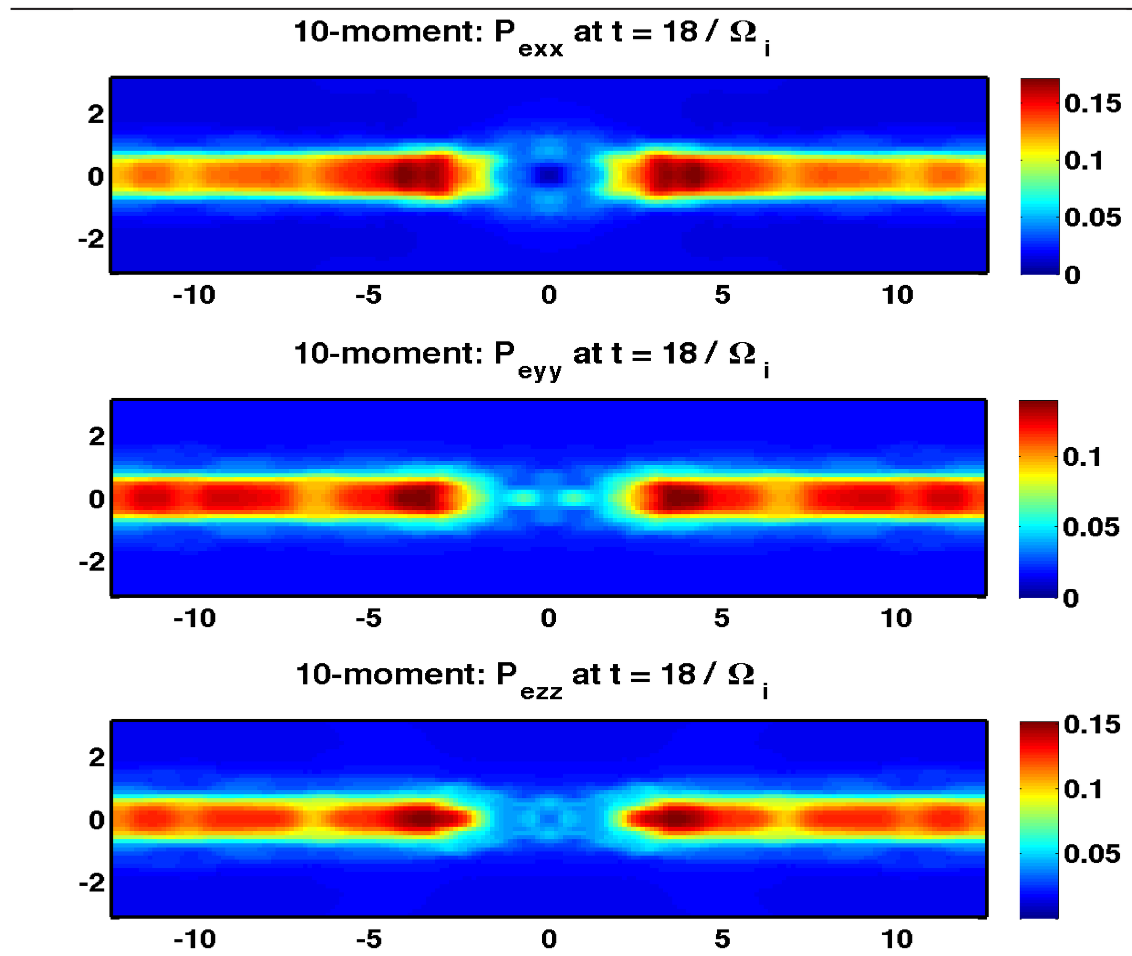
Vlasov, quoted from **[ScGr06]**

Times for each model at which the magnetic flux through the positive x-axis reached one nondimensionalized unit:

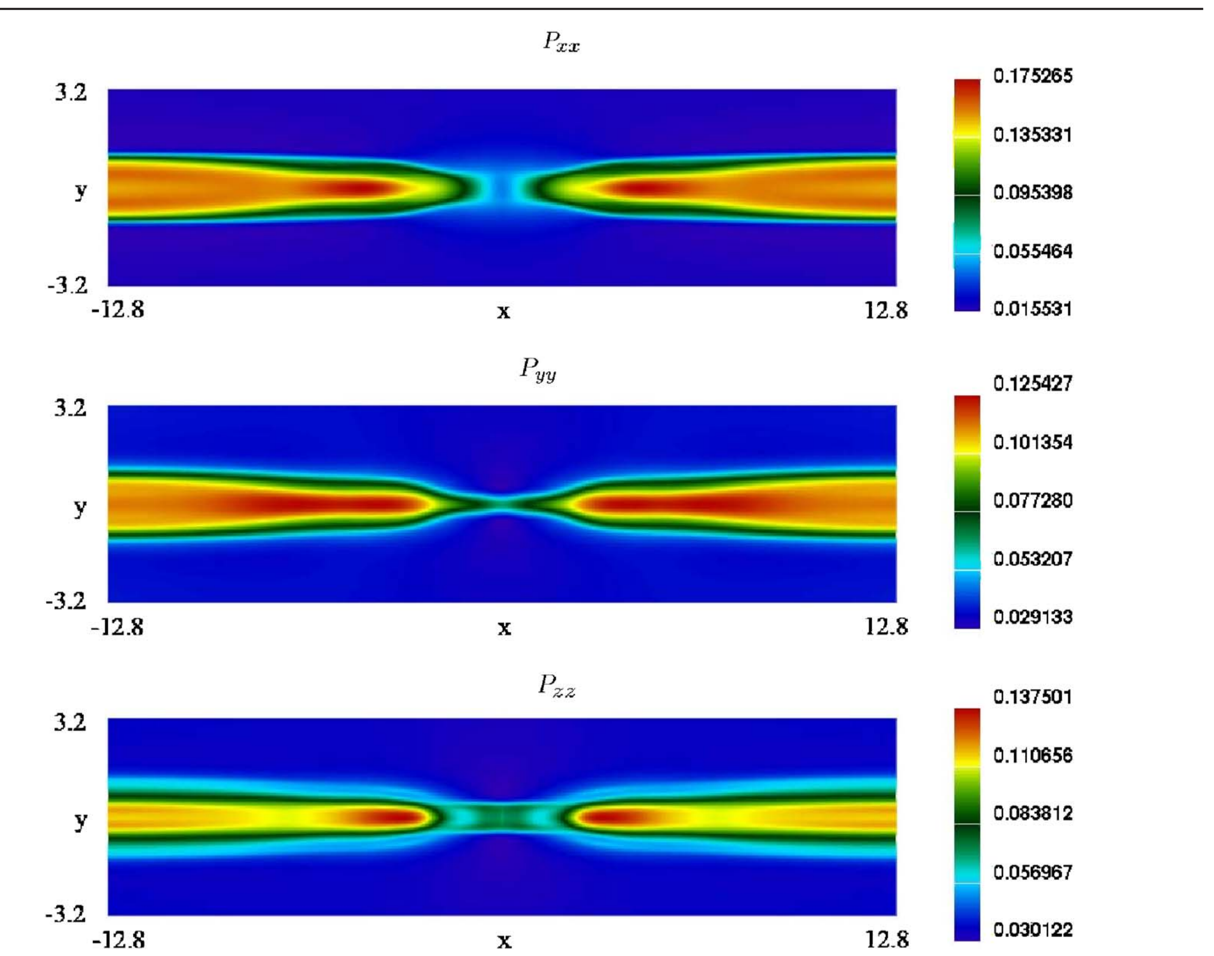
model	time	study
Vlasov	$17.7/\Omega_i$	[ScGr06]
PIC	$15.7/\Omega_i$	[Pritchett01]
10-moment	$\mathbf{17.2/\Omega_i}$	**[JoRo10]**
10/5-moment	$17.6/\Omega_i$	[Hakim07]
5-moment	$15.6/\Omega_i$	[HaLoShu06]
5-moment	$15.3/\Omega_i$	[LoHaShu10]
5-moment	$\mathbf{13.0/\Omega_i}$	**[JoRo10]**

Electron pressure tensor

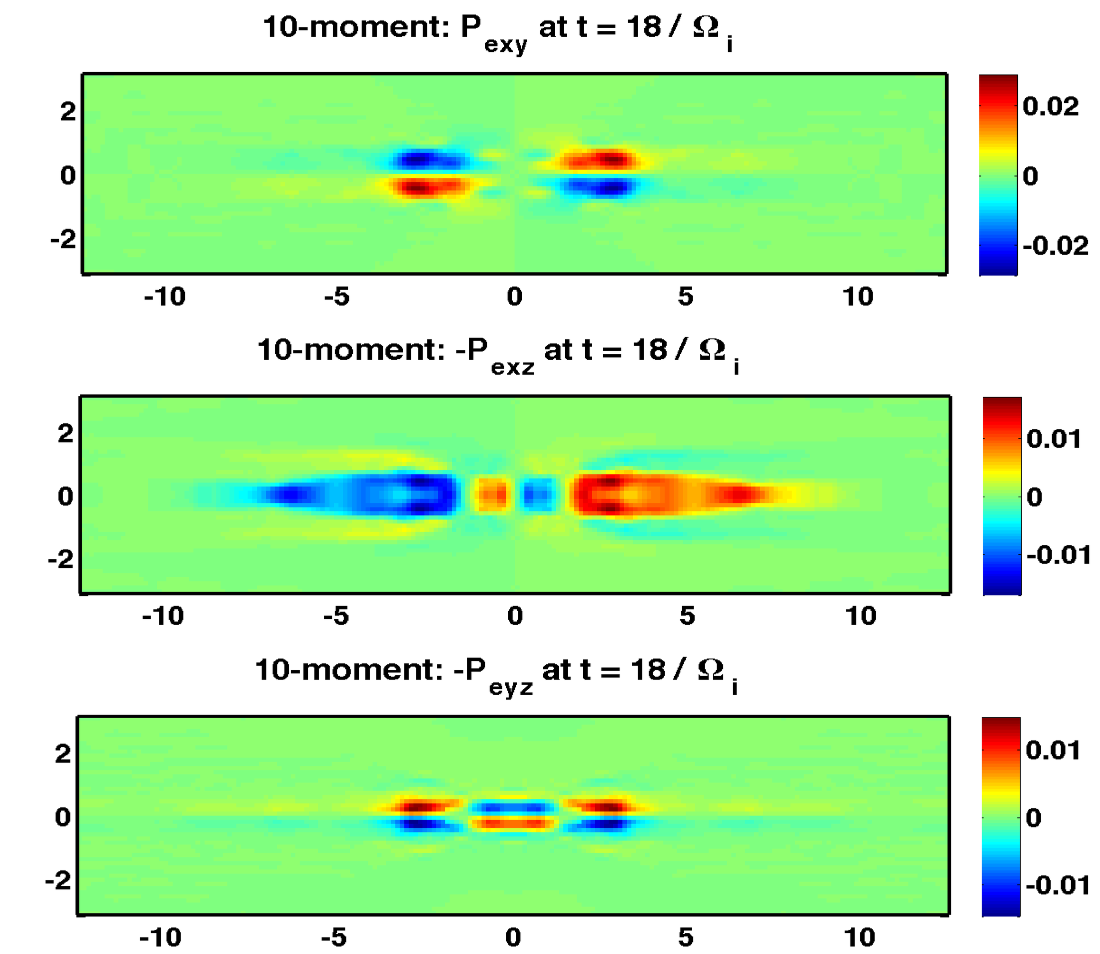
The 10 moment model gets Ohm's law right because it resolves off-diagonal pressure tensor components.



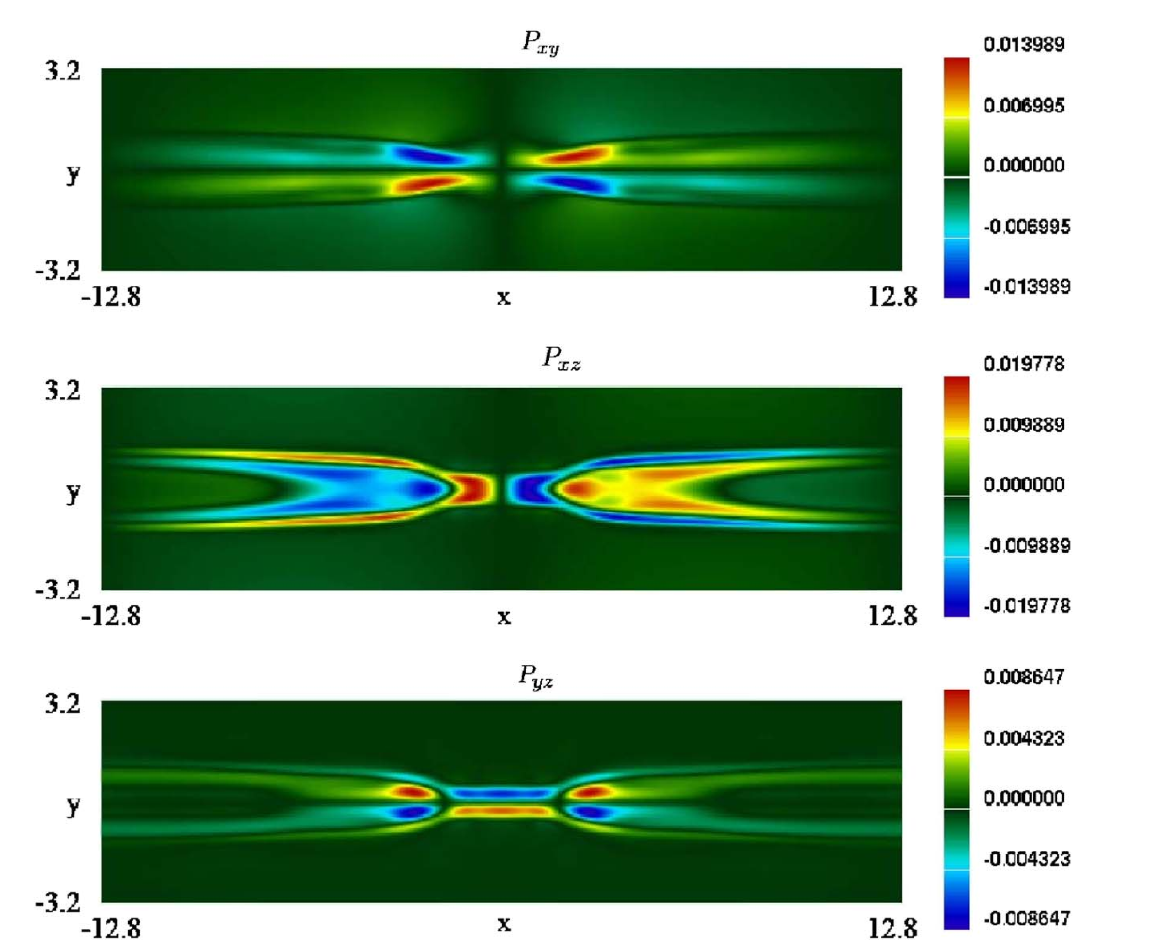
Diagonal components of the electron pressure tensor for our **10-moment** simulation at $\Omega_i t = 18$



Schmitz and Grauer's diagonal components for **Vlasov** simulation at $\Omega_i t = 17.7$ [ScGr06]



Off-diagonal components of the electron pressure tensor for our **10-moment** simulation at $\Omega_i t = 18$



Schmitz and Grauer's off-diagonal components for **Vlasov** simulation at $\Omega_i t = 17.7$ [ScGr06]

References

[JoRo10] E. A. Johnson and J. A. Rossmanith, Ten-moment two-fluid model agrees well with PIC/Vlasov in GEM problem, submitted to proceedings of HYP2010 (2010).

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