Fast Magnetic Reconnection in Fluid Models of (Pair) Plasma

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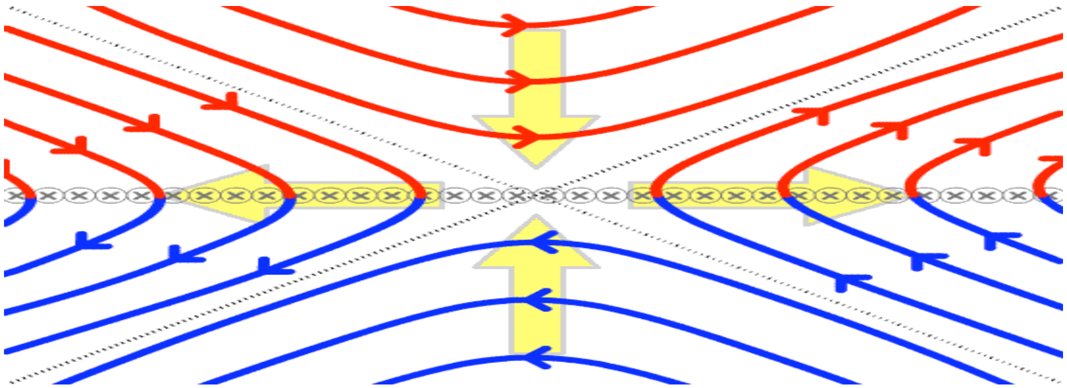


Outline

① Magnetic Reconnection

② GEM magnetic reconnection problem

③ pair plasma reconnection

④ deficient fluid models
   (a) magnetized
   (b) anomalous resistivity
   (c) adiabatic five-moment
   (d) adiabatic ten-moment

⑤ intermediate isotropization model

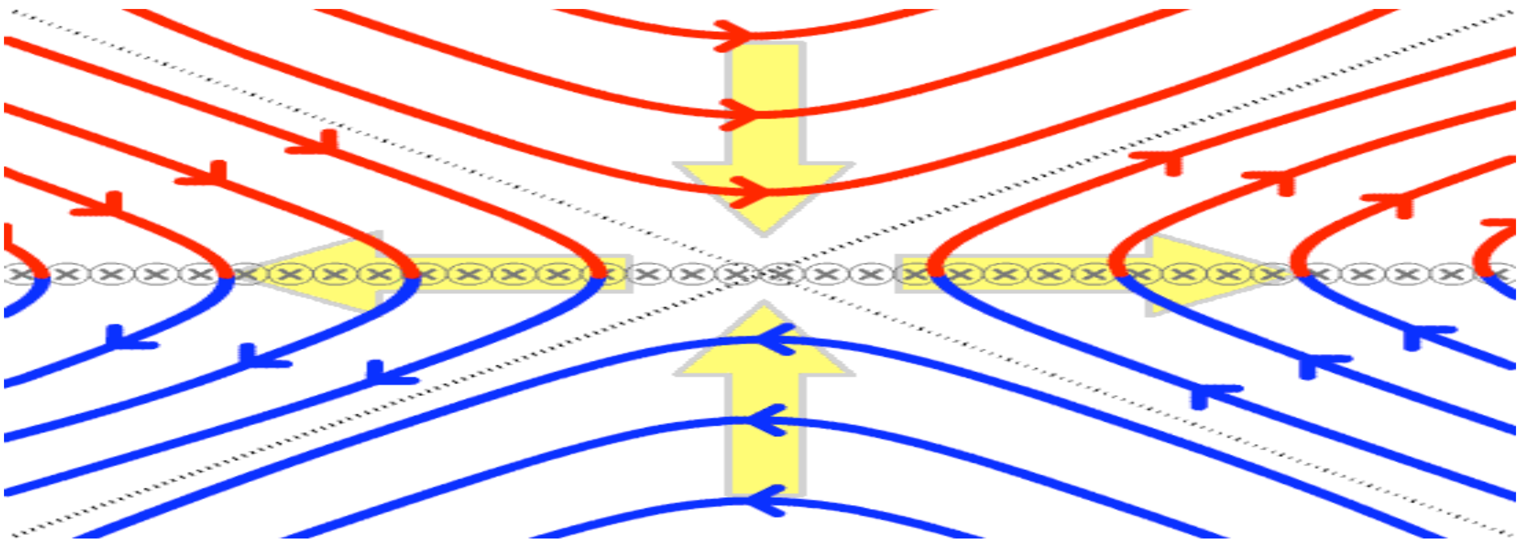
Claim: We can get fast reconnection near a magnetic null point using a fluid model (of pair plasma) without anomalous resistivity.

Process of Magnetic Reconnection

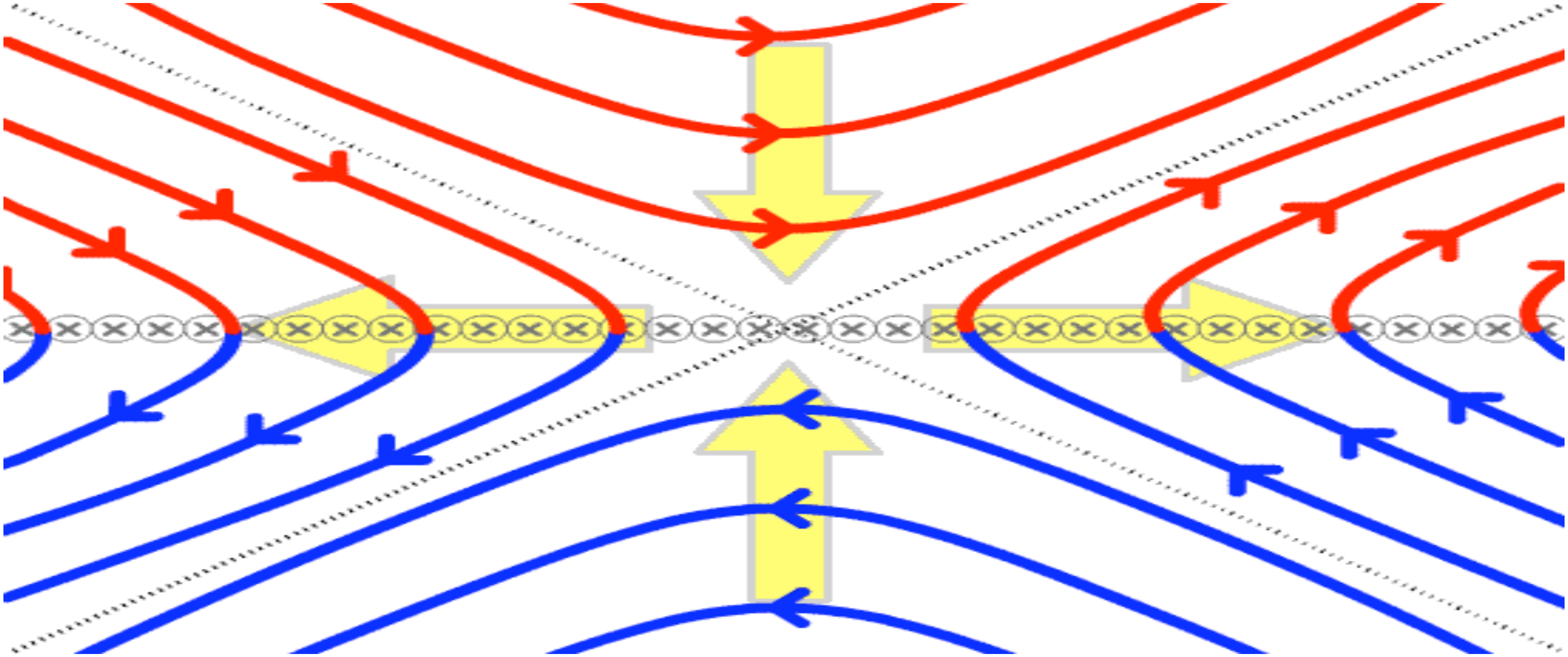
Plasma: gas of charged particles, carries magnetic field.

Field lines are convected with plasma except near reconnection points.

Adjacent oppositely directed magnetic field lines field lines come together and cancel and reconnect, converting magnetic energy into kinetic energy.

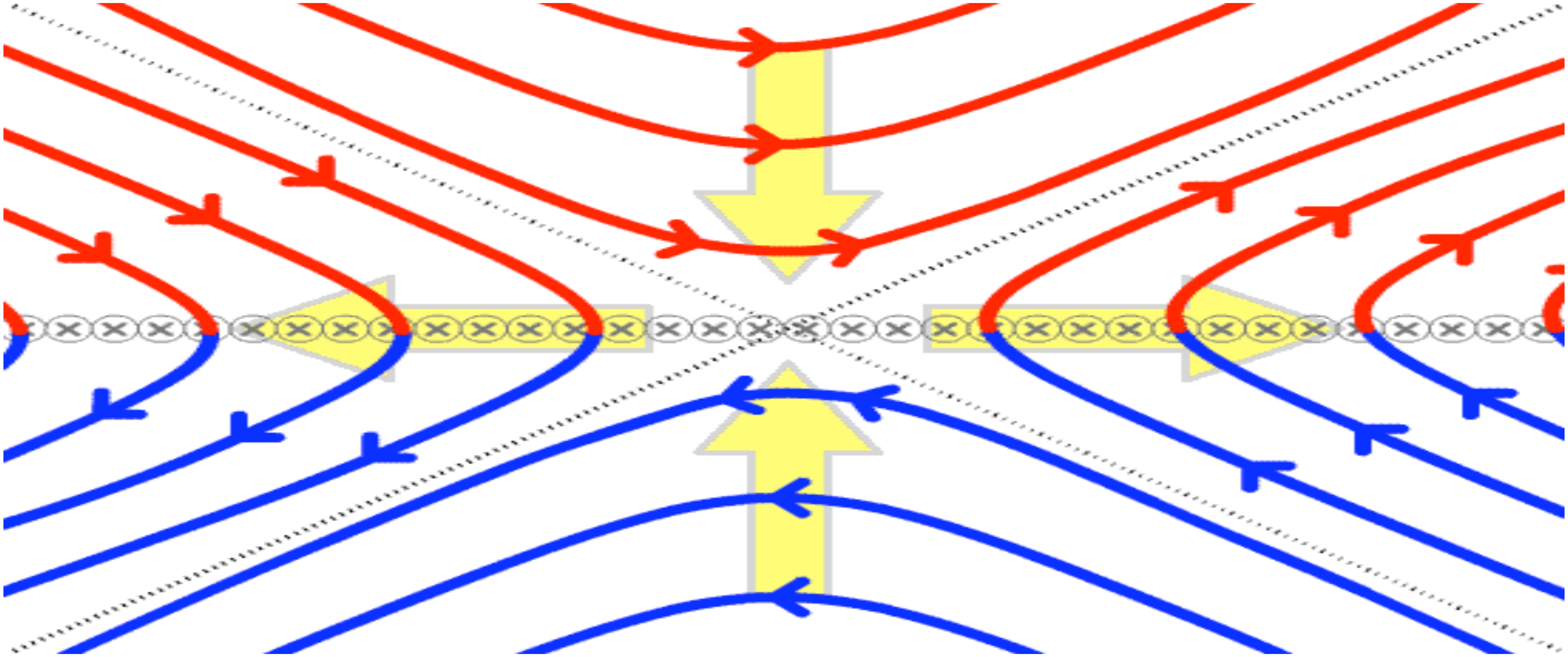


Magnetic Reconnection



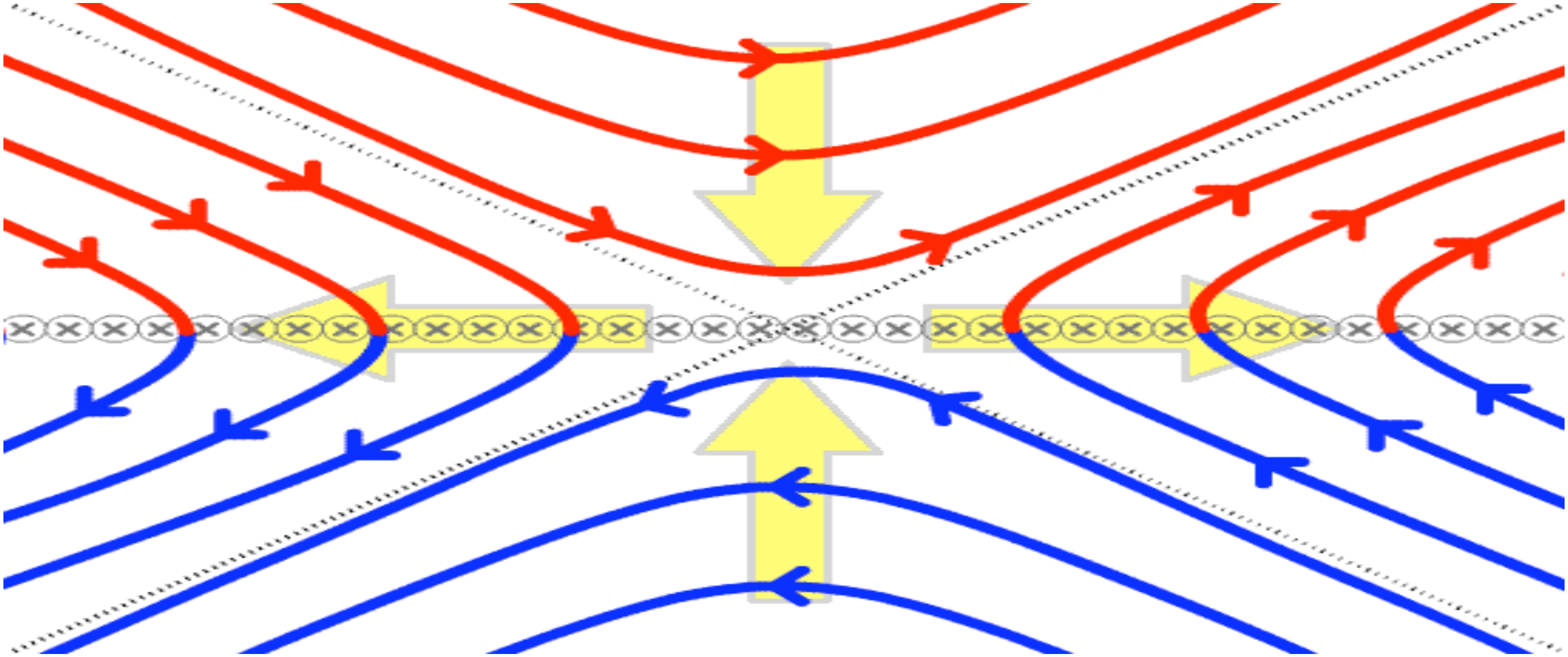
(wikipedia)

Magnetic Reconnection



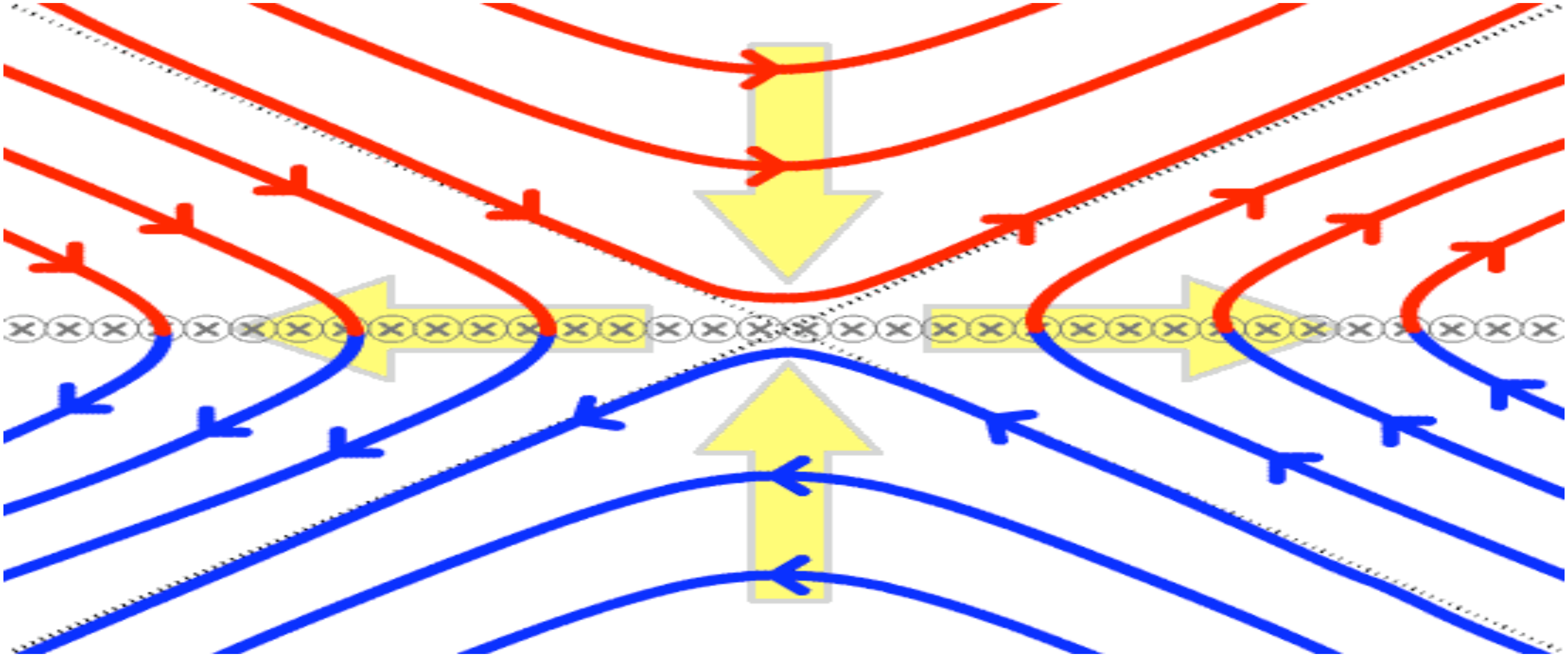
(wikipedia)

Magnetic Reconnection



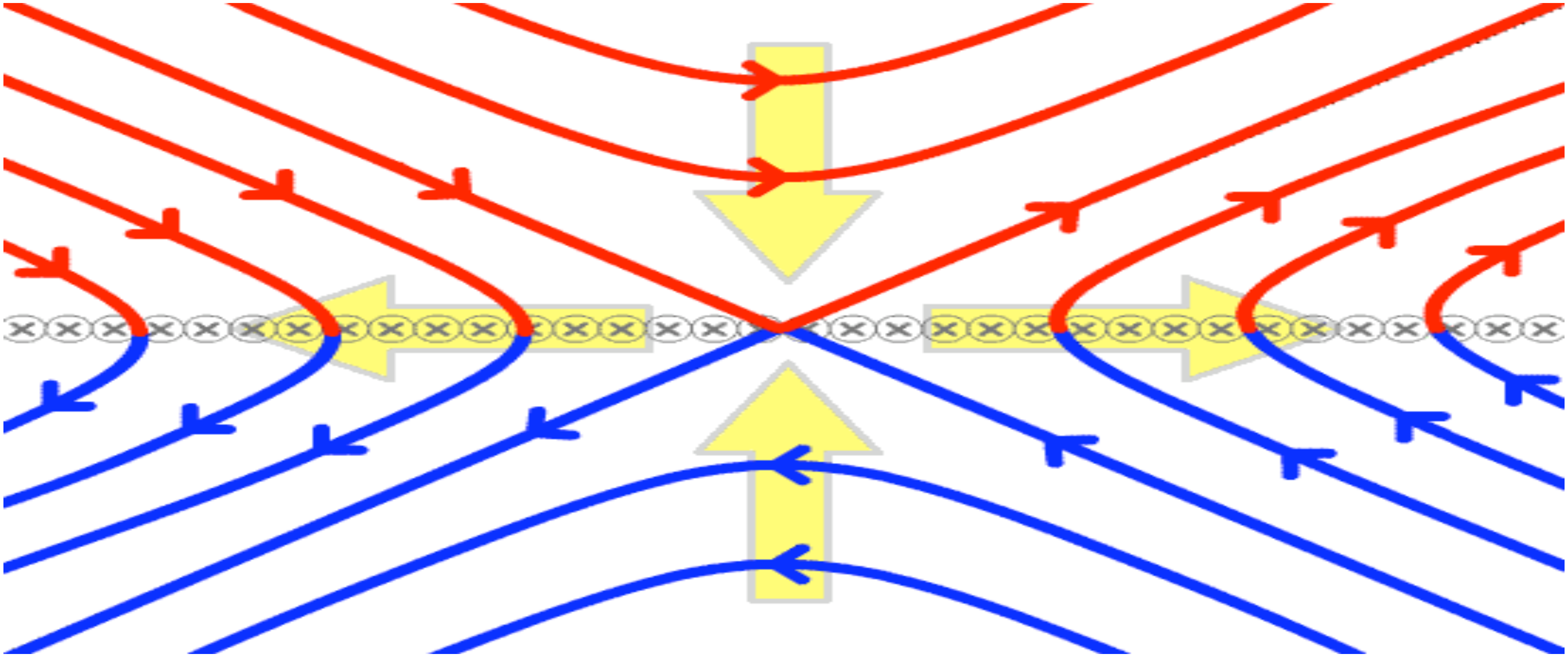
(wikipedia)

Magnetic Reconnection



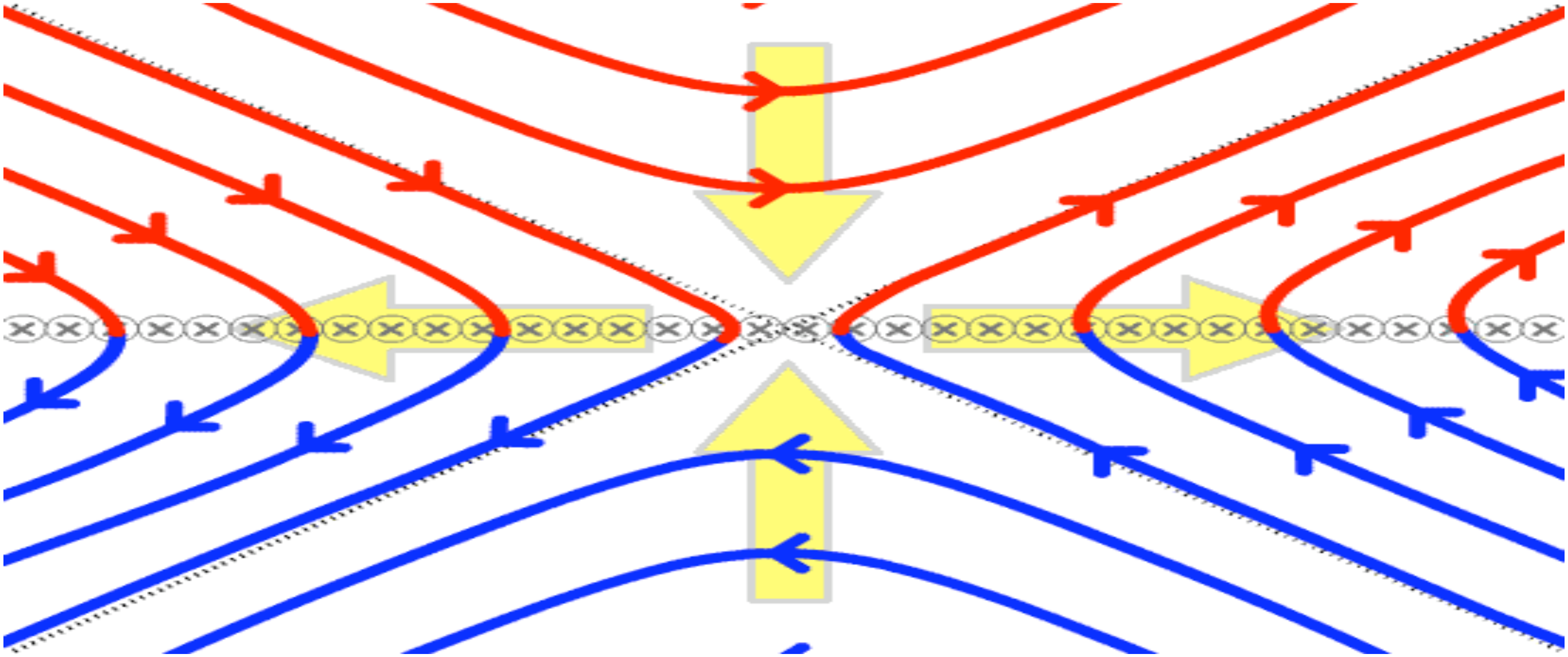
(wikipedia)

Magnetic Reconnection



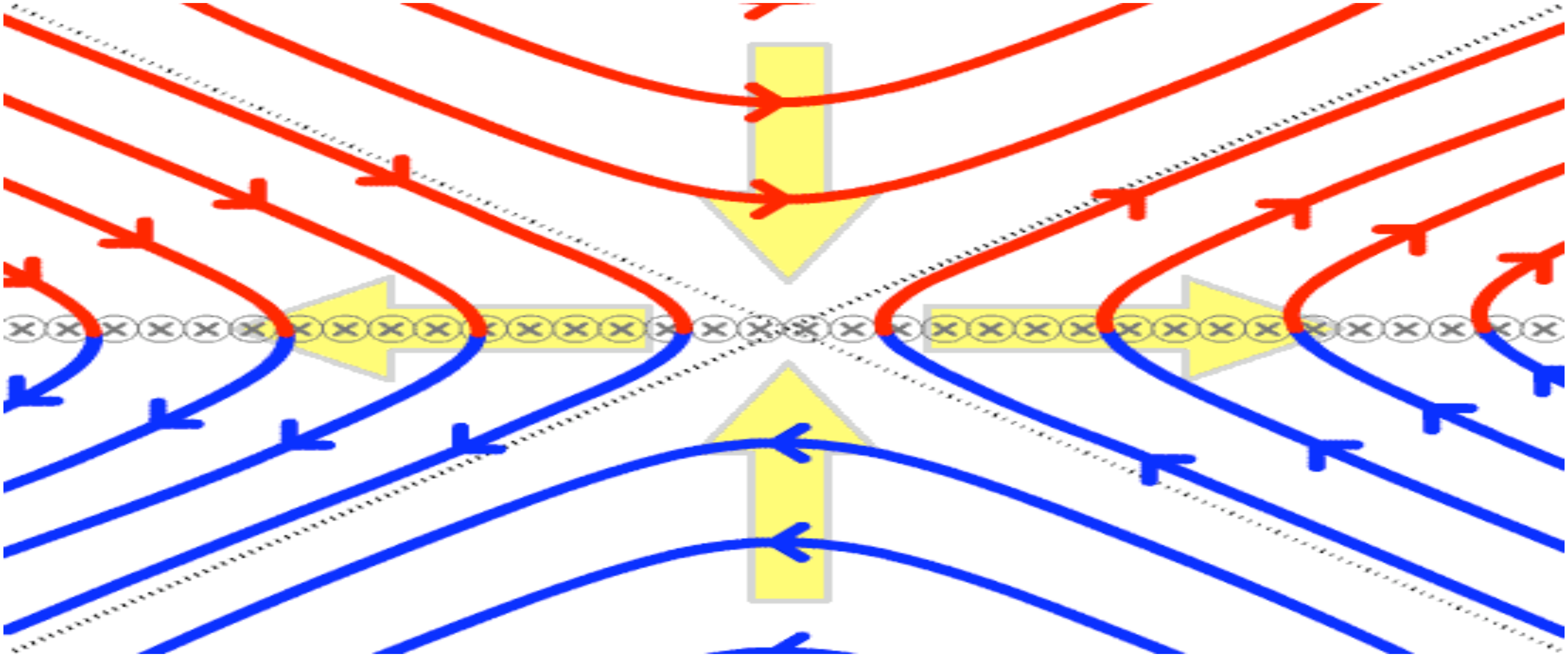
(wikipedia)

Magnetic Reconnection



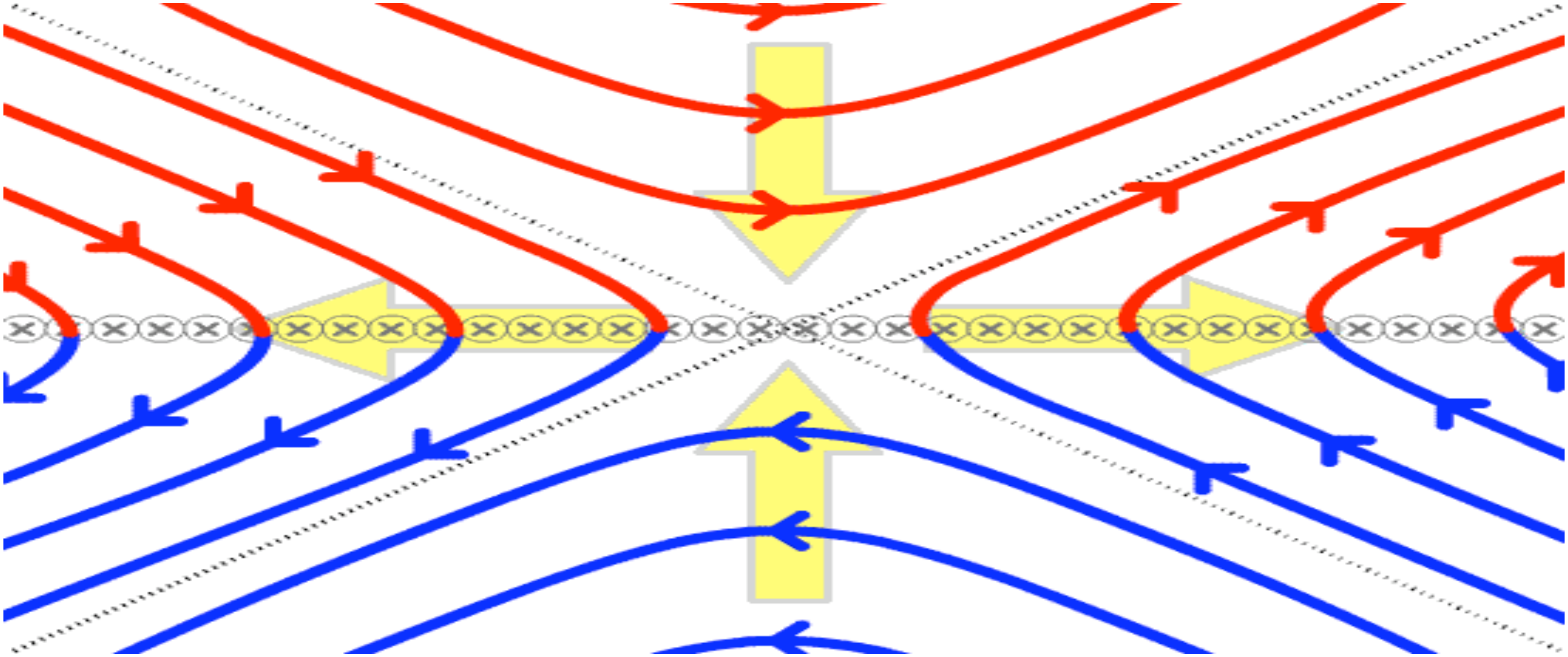
(wikipedia)

Magnetic Reconnection



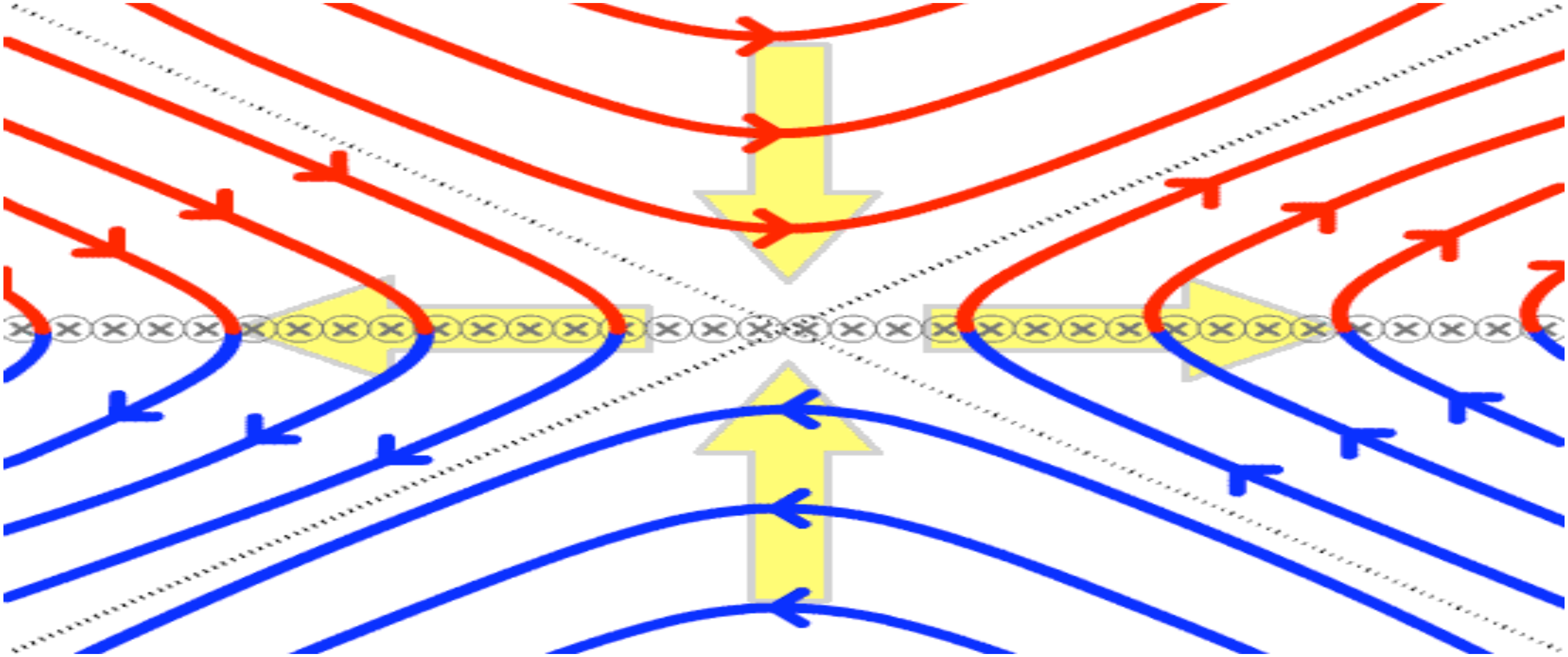
(wikipedia)

Magnetic Reconnection



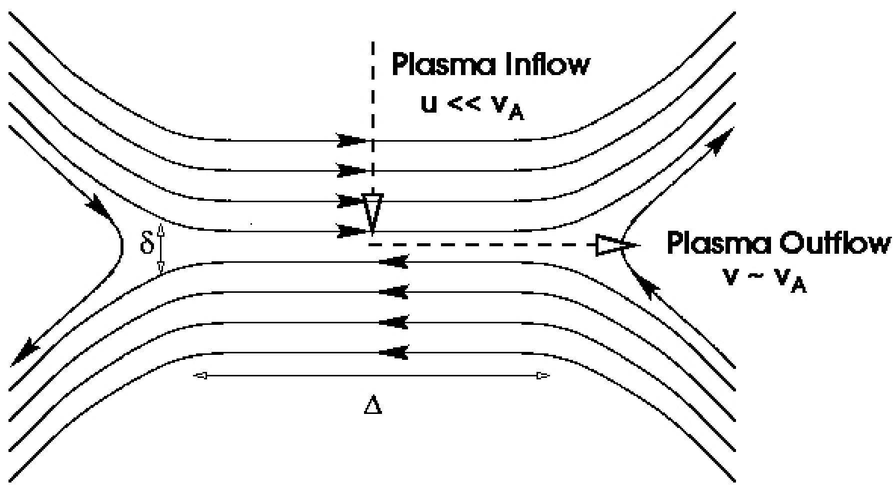
(wikipedia)

Magnetic Reconnection



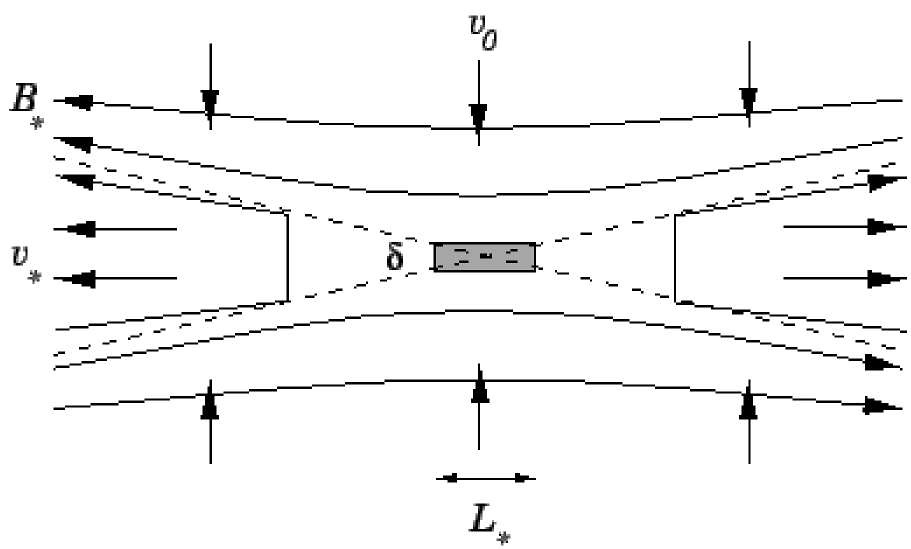
(wikipedia)

Slow magnetic reconnection (Sweet-Parker model)



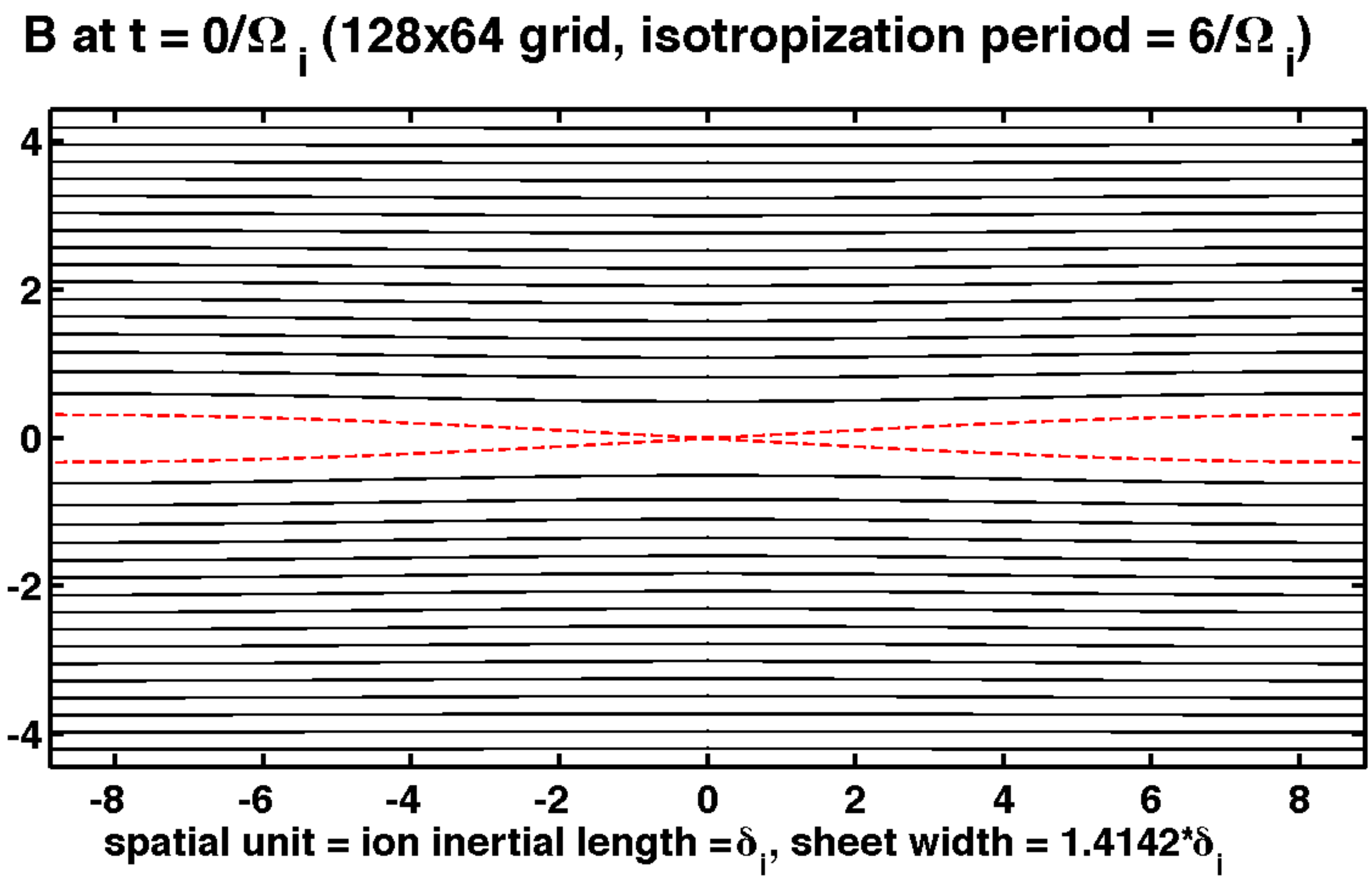
http://mrx.pppl.gov/Physics/physics.html

Fast magnetic reconnection (Petschek model)

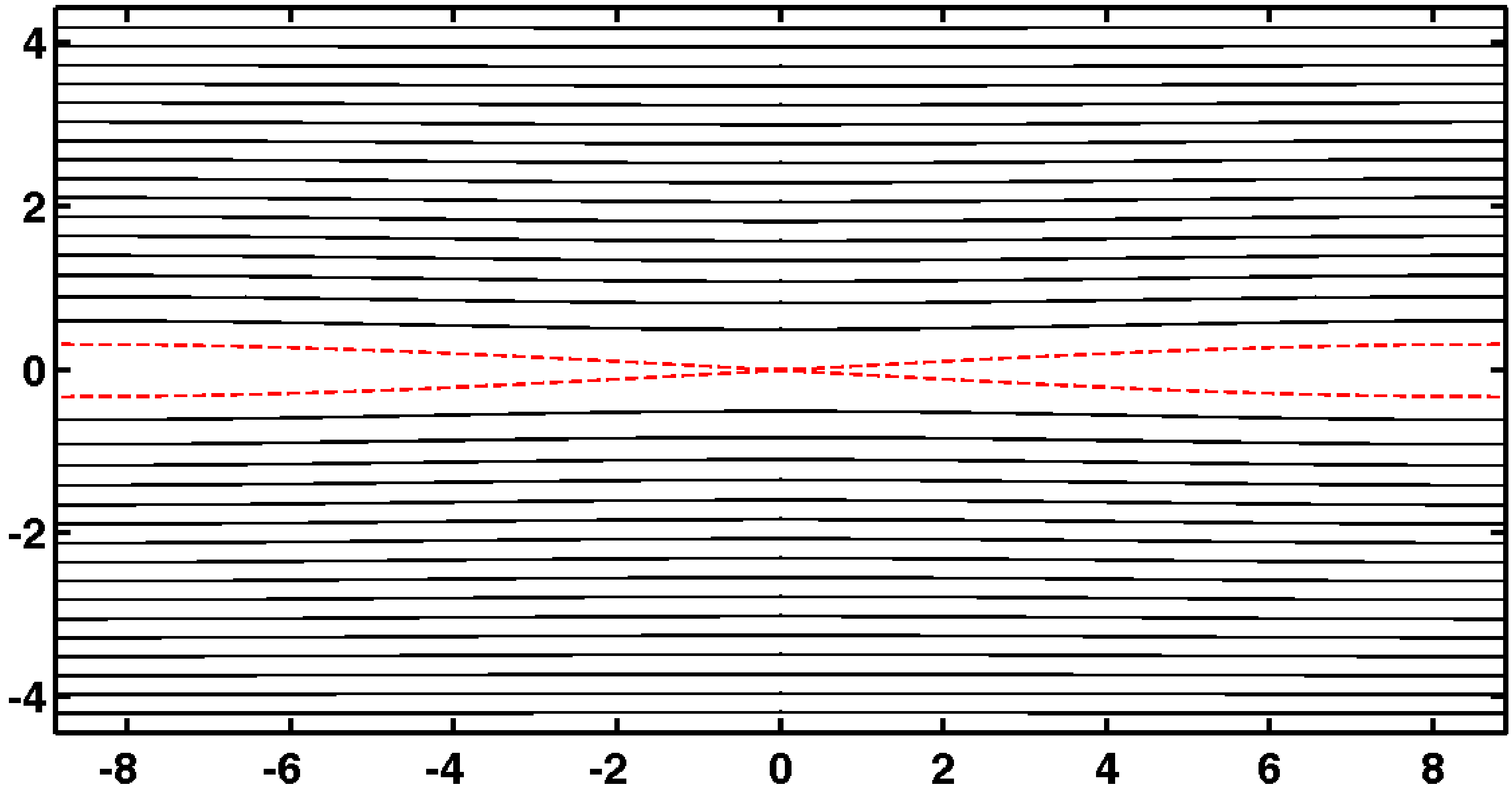


http://farside.ph.utexas.edu/teaching/plasma/lectures/node77.html

Magnetic reconnection: GEM problem

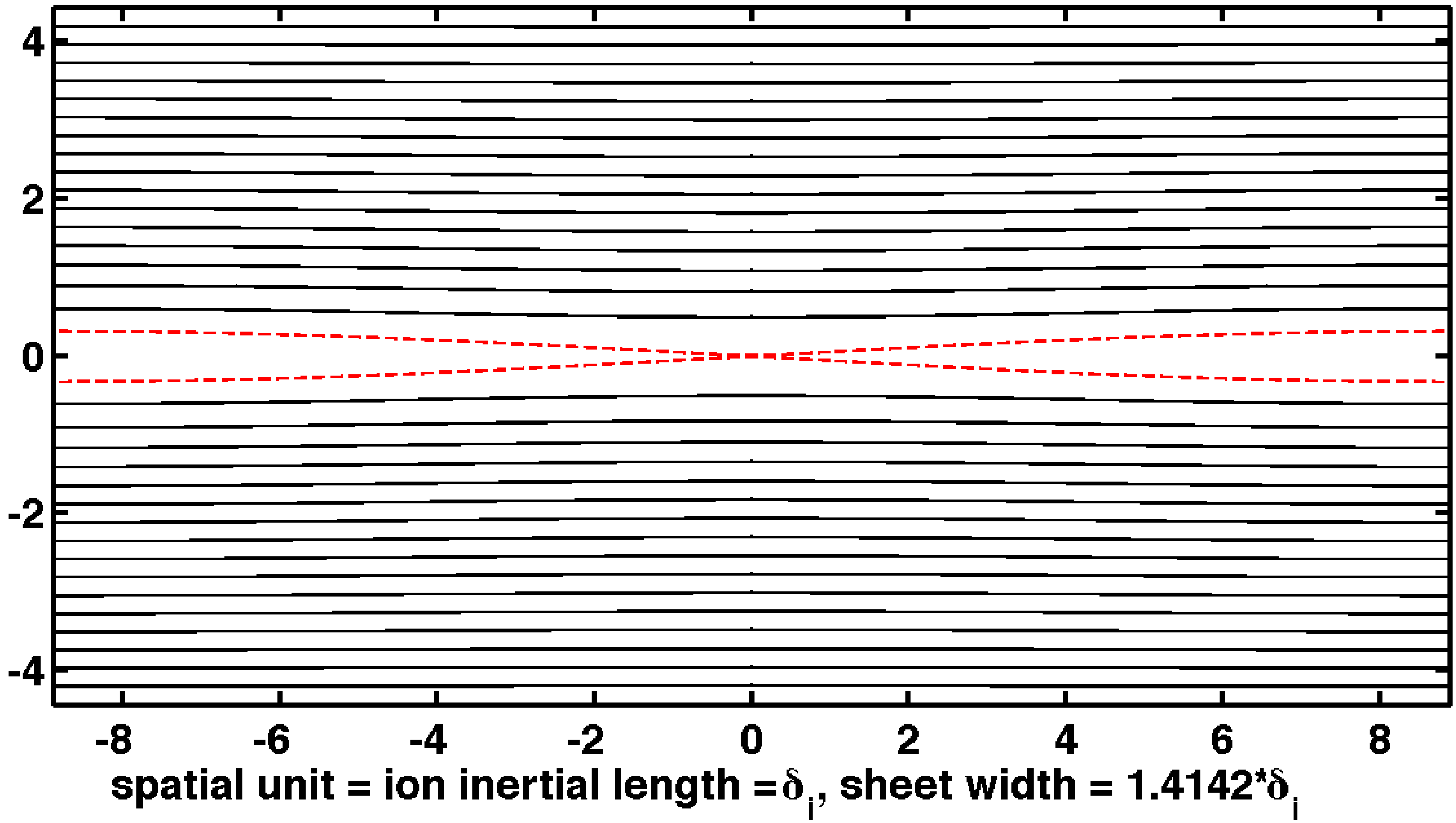


B at t = 0/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

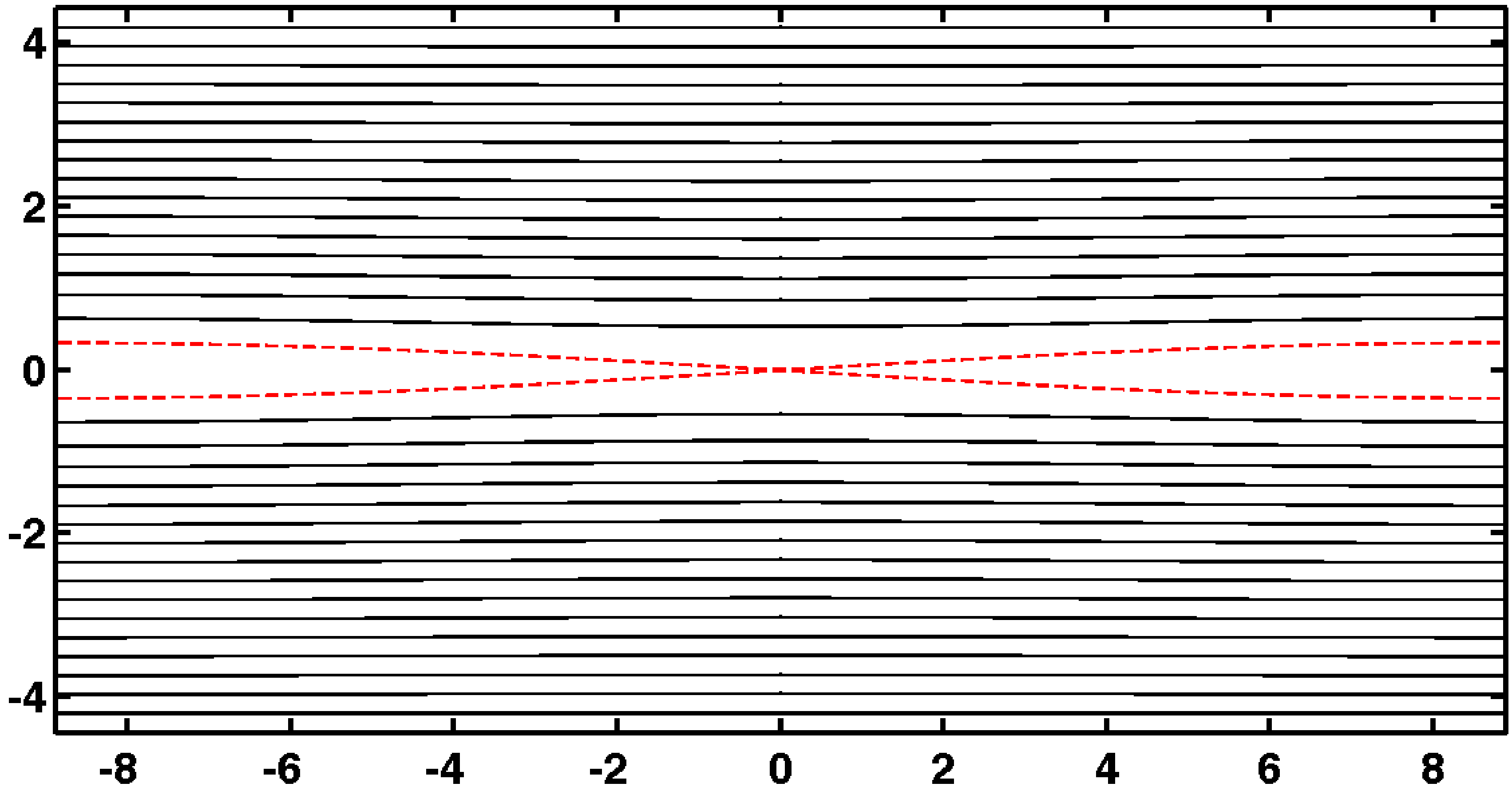


spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

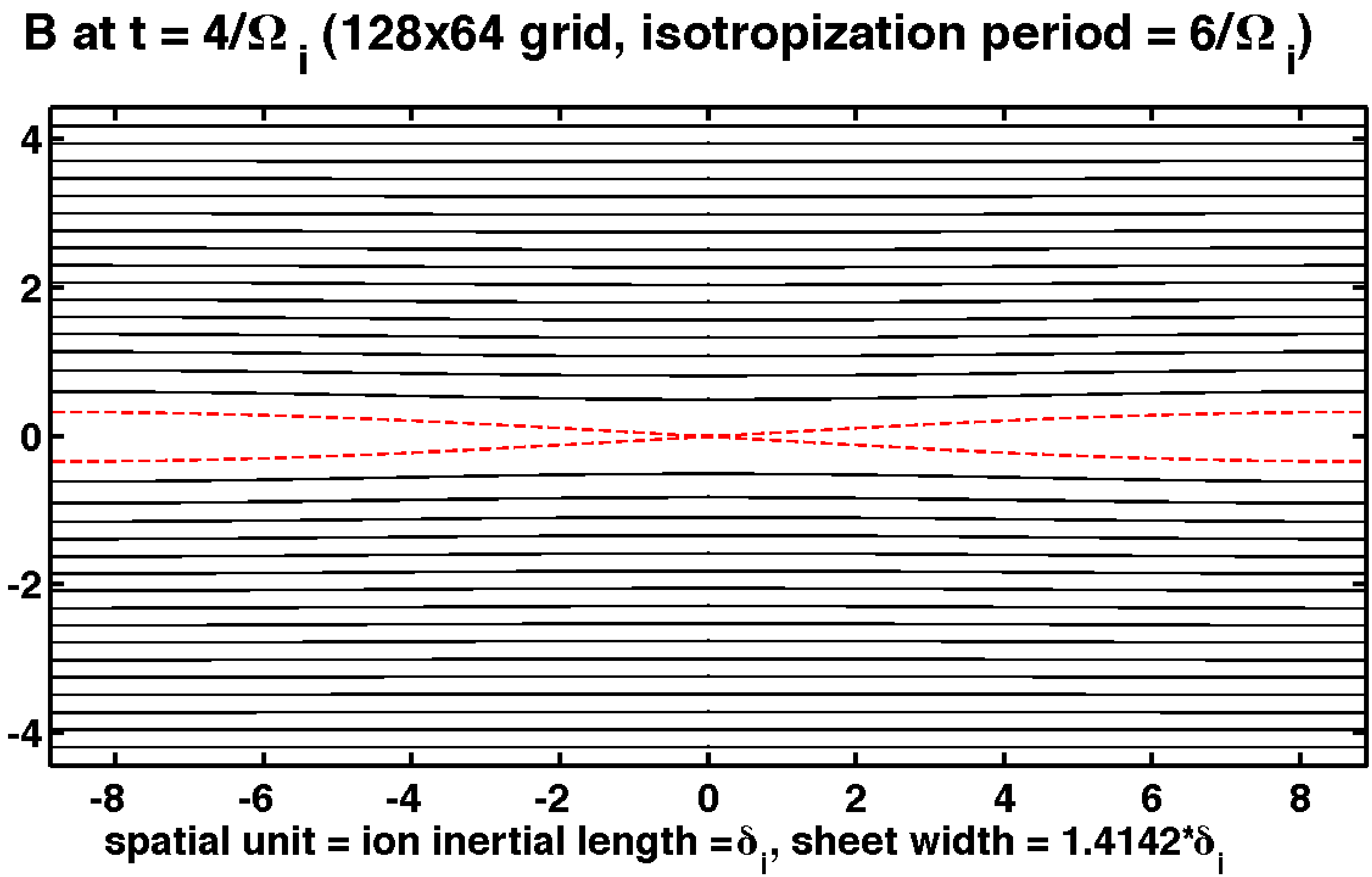
B at t = 0/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



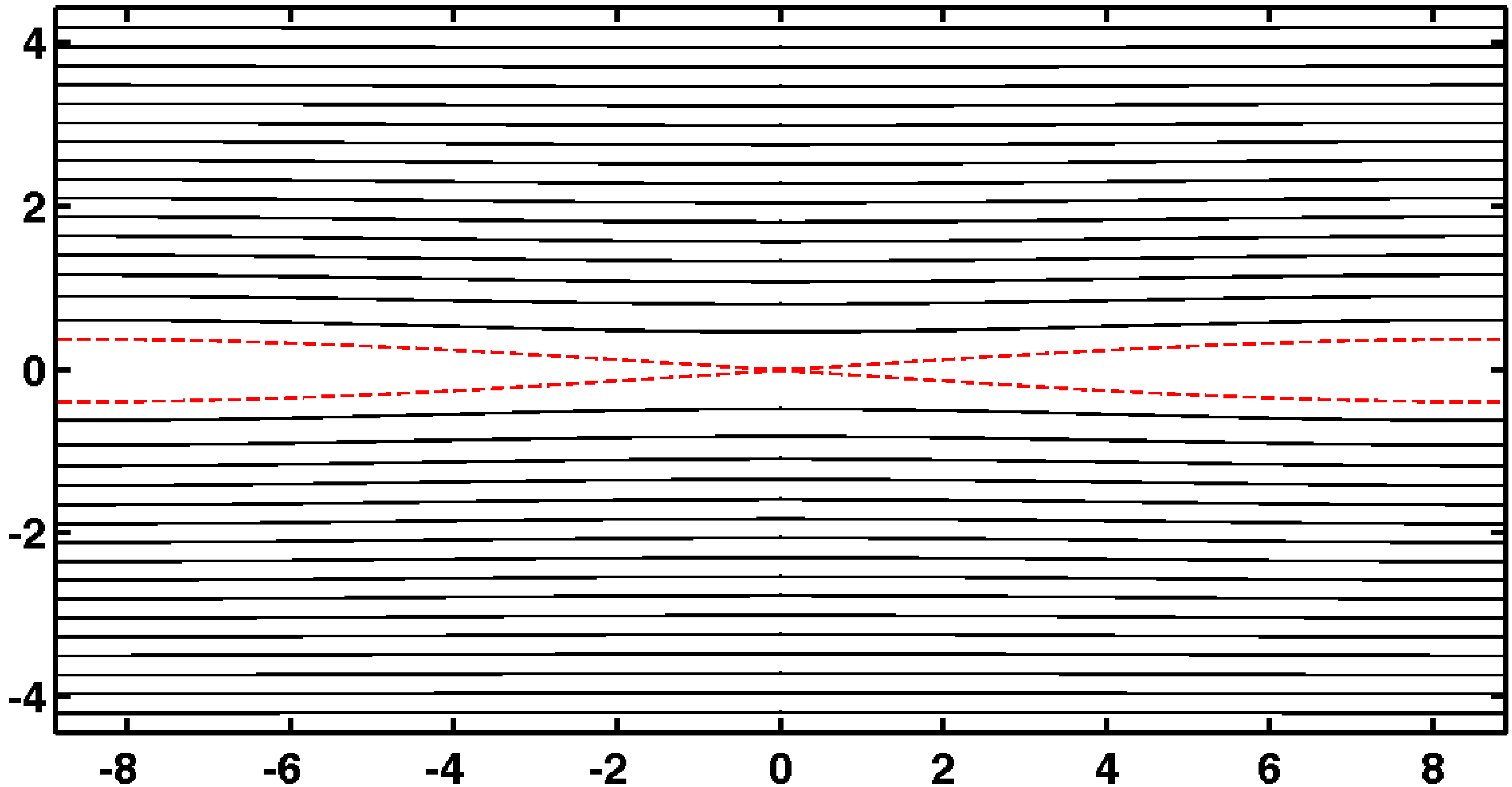
B at t = $2/\Omega_i$ (128x64 grid, isotropization period = $6/\Omega_i$)



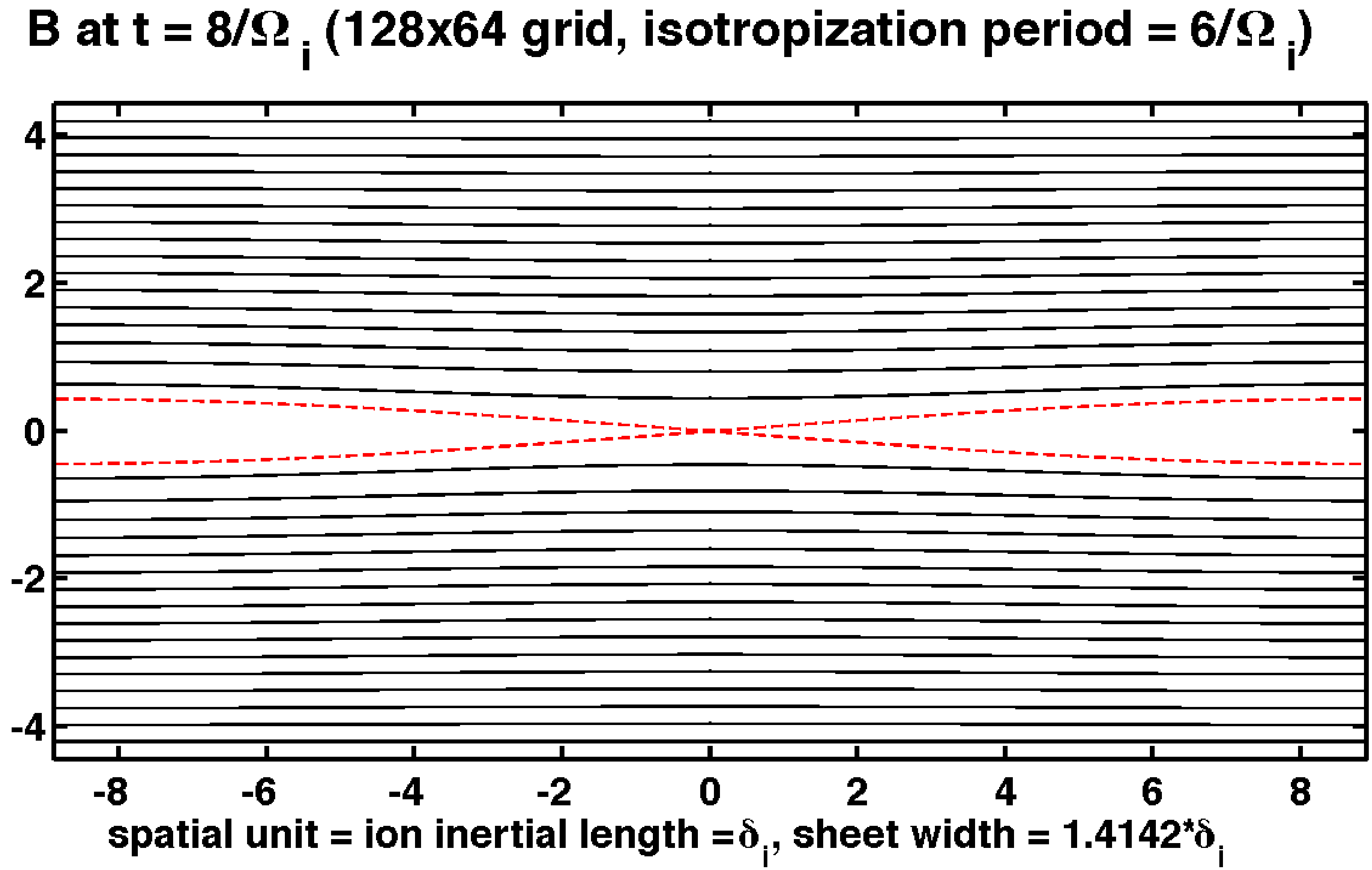
spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i


B at t = 4/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

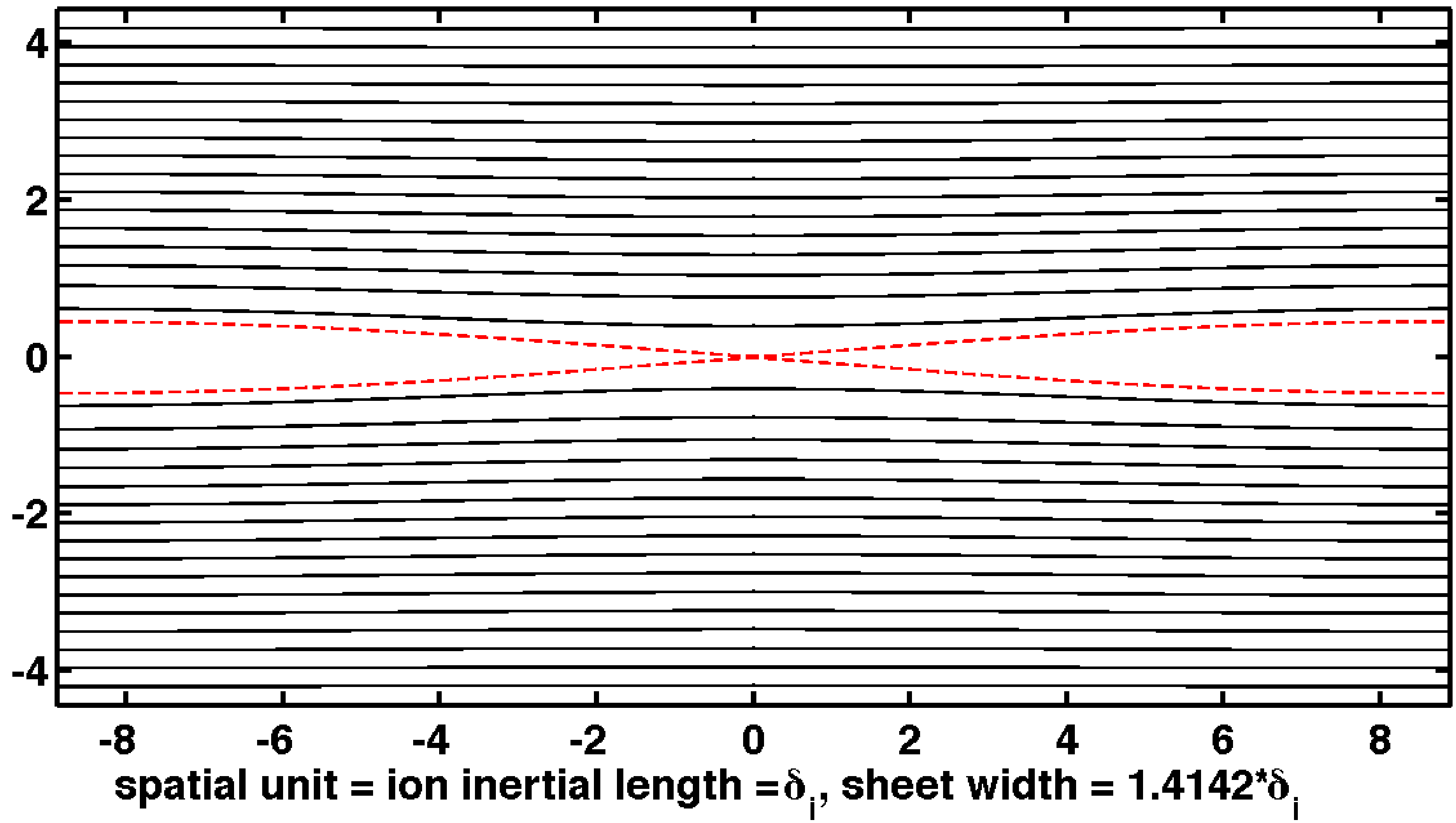
B at t = 6/Ω i (128x64 grid, isotropization period = 6/Ω i)



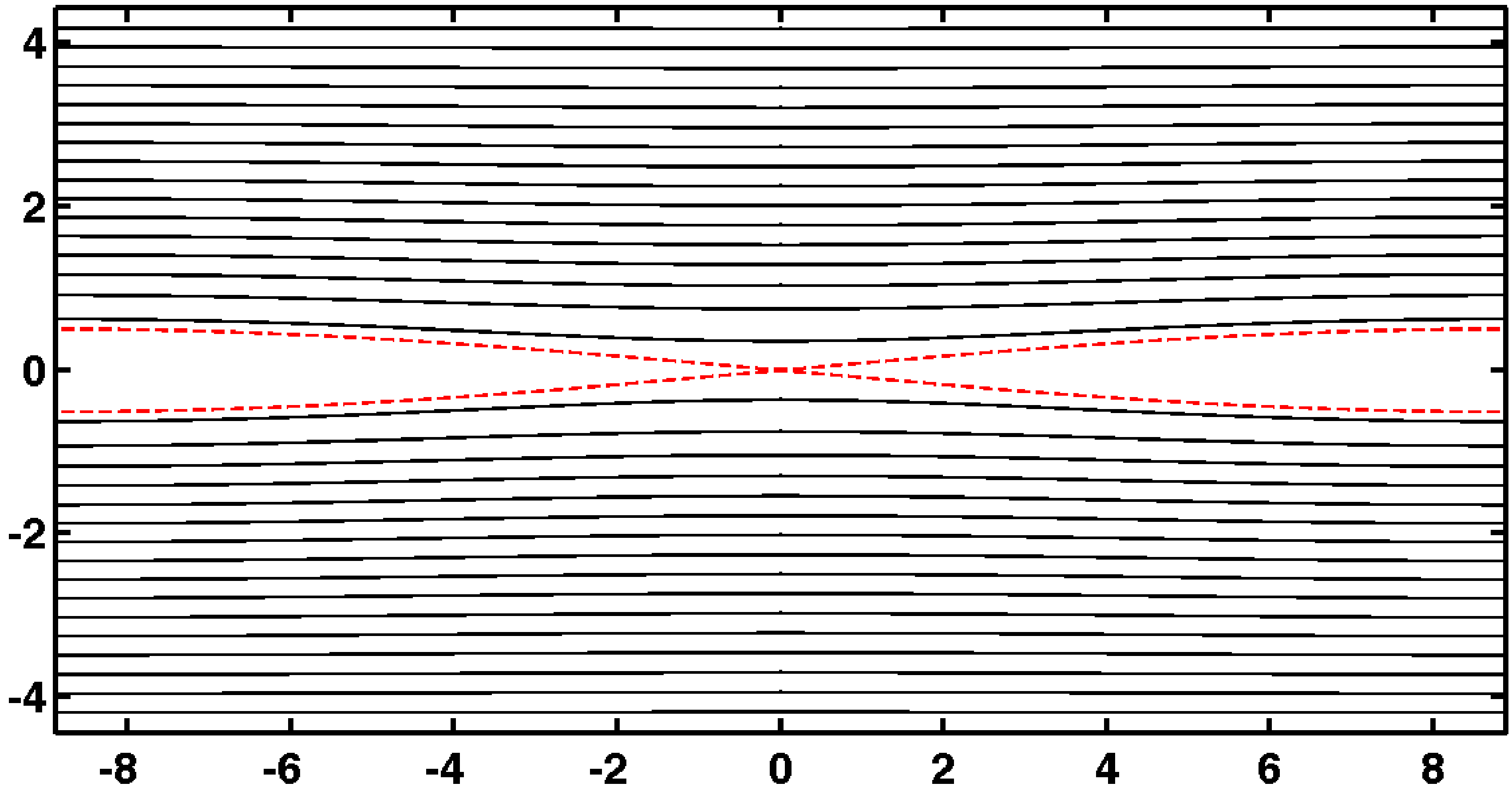
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 8/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

B at t = 10/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

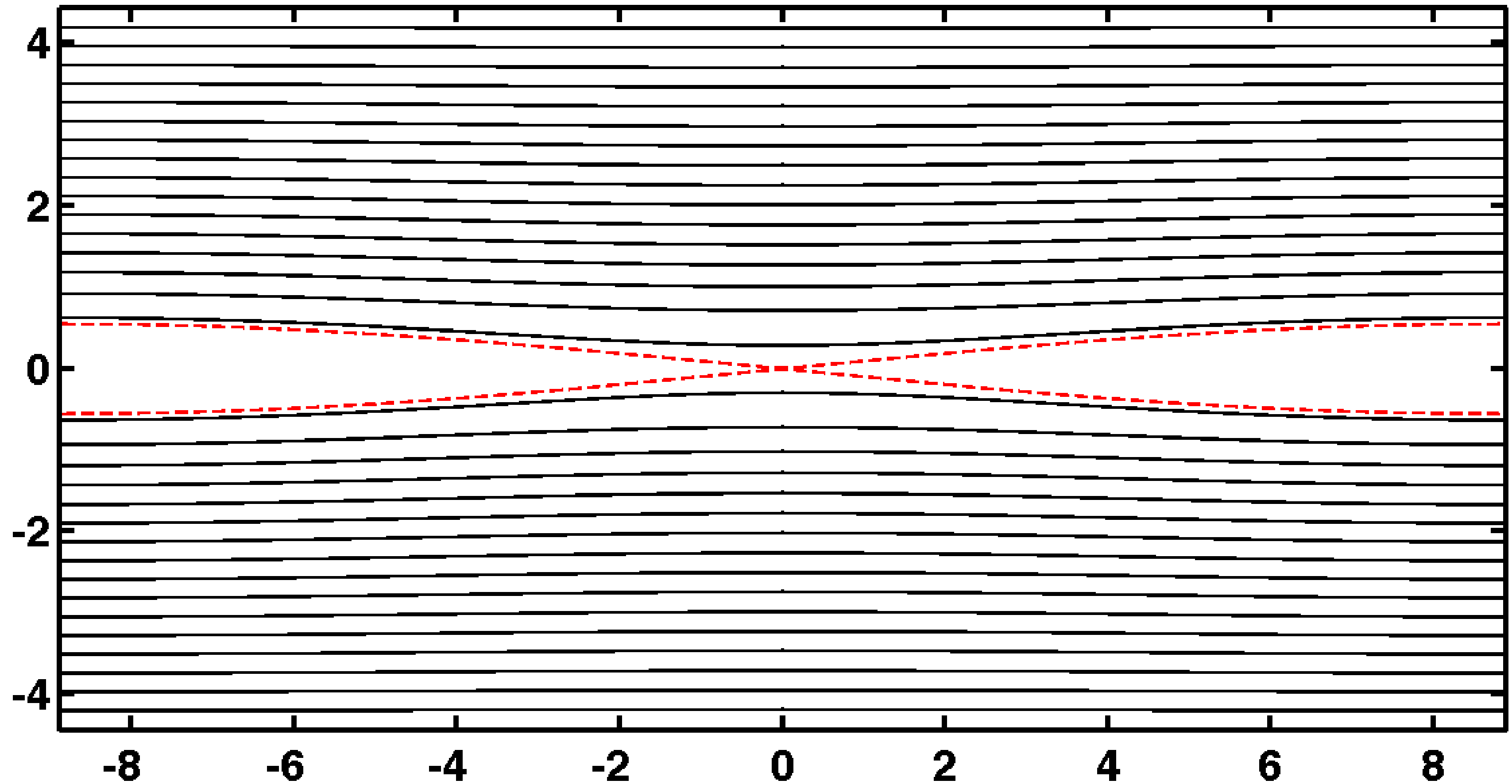


B at t = 12/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



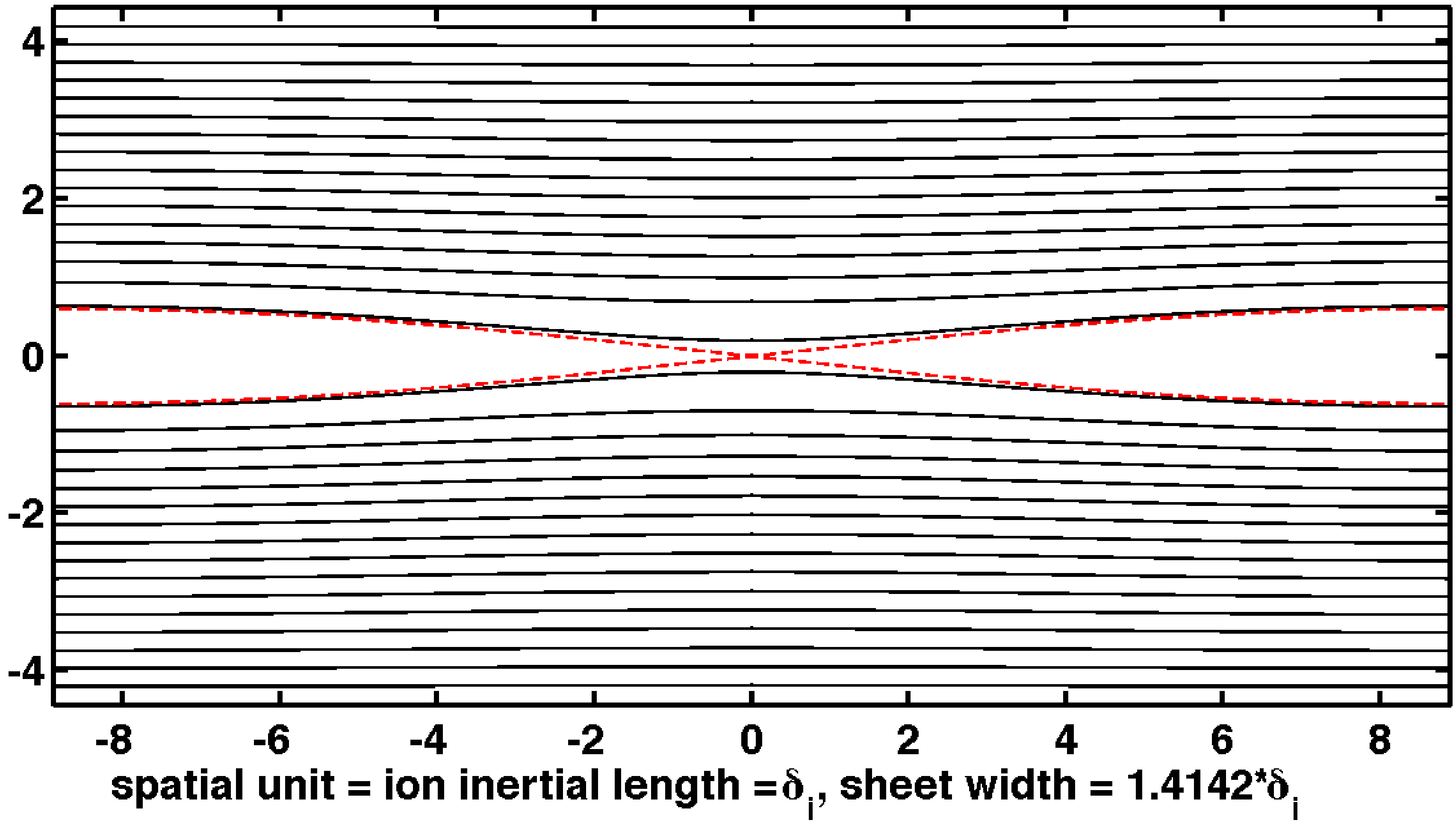
spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i

B at t = $14/\Omega_i$ (128x64 grid, isotropization period = $6/\Omega_i$)

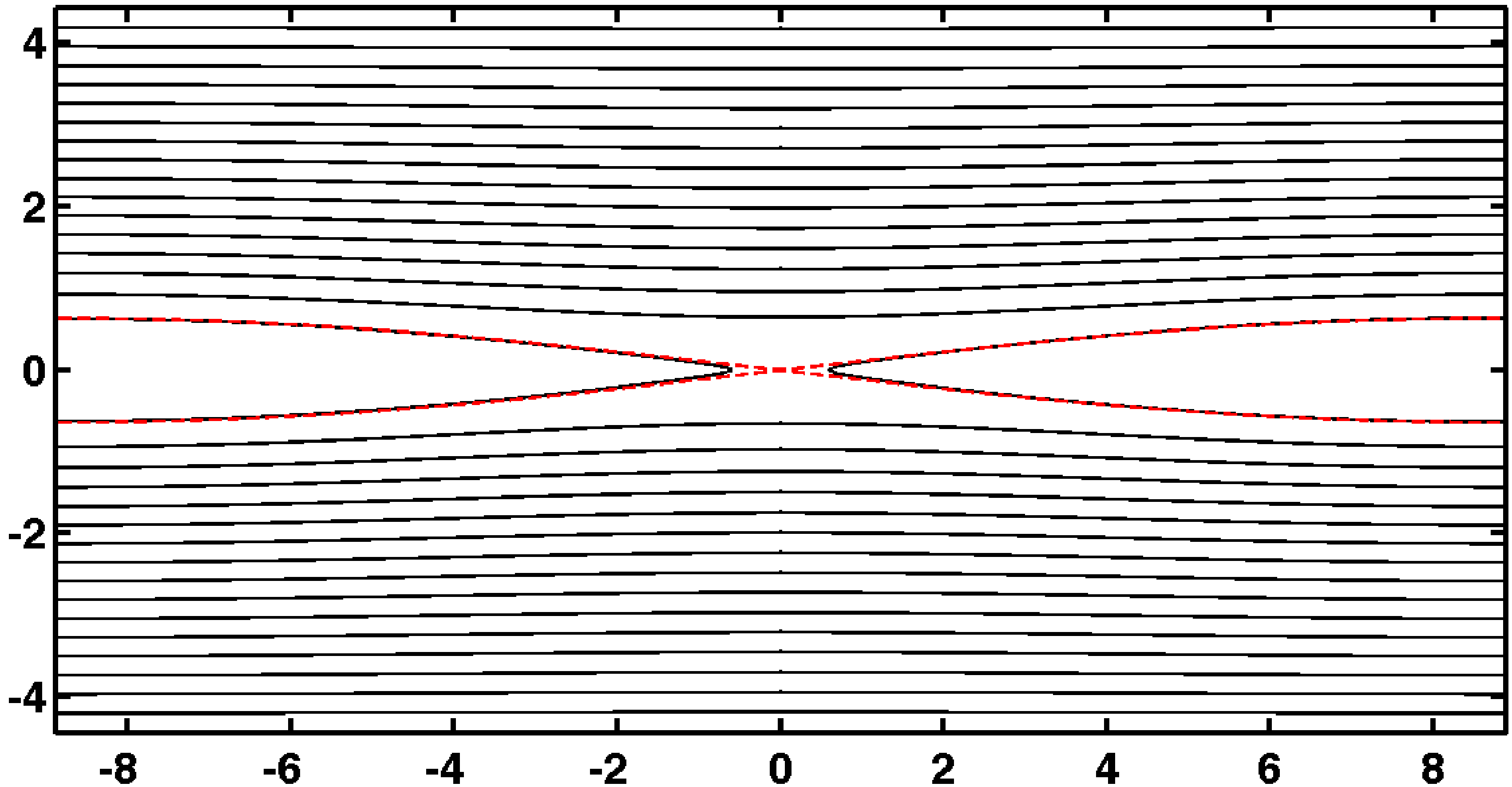


spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

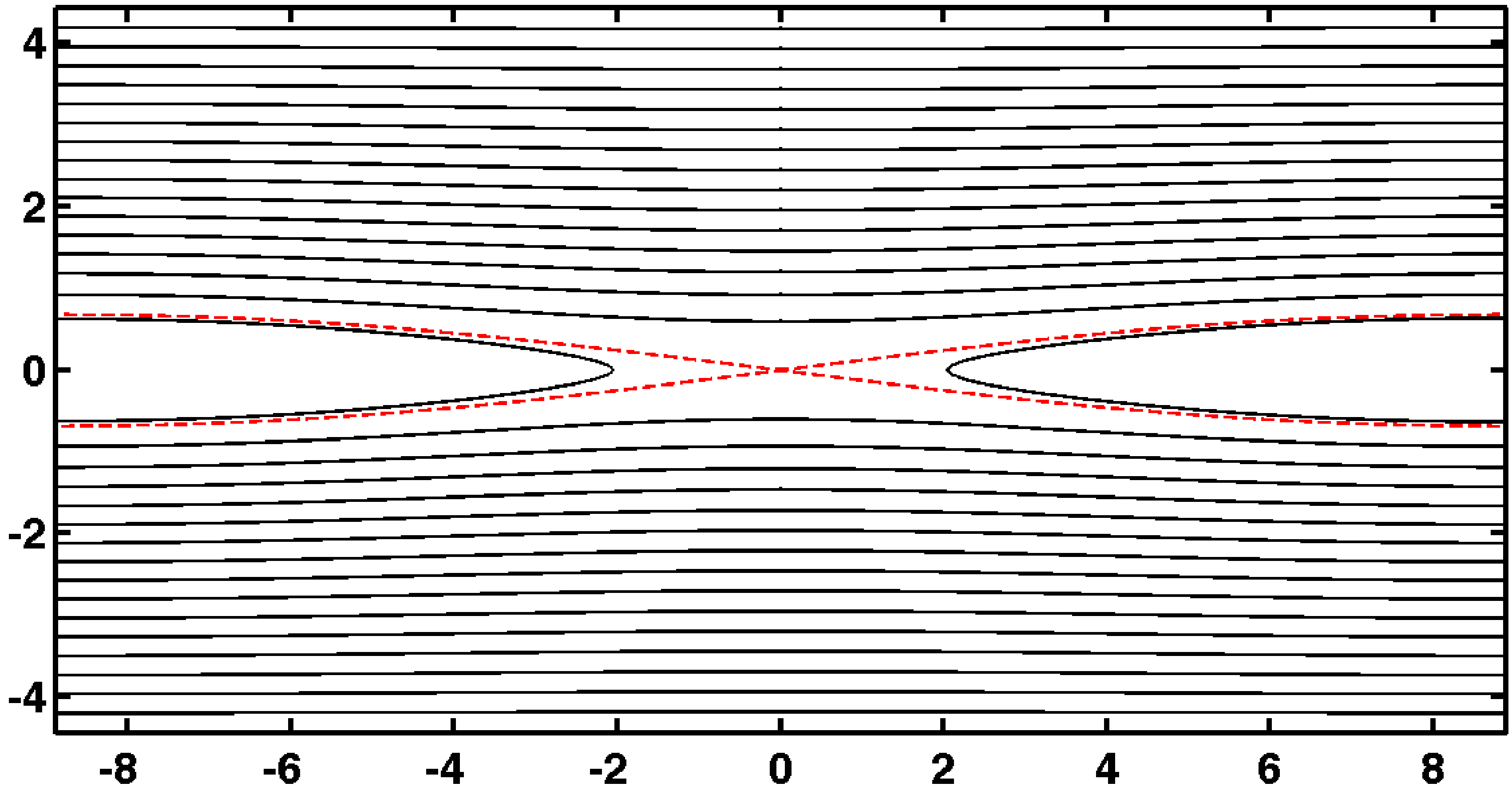
B at t = 16/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



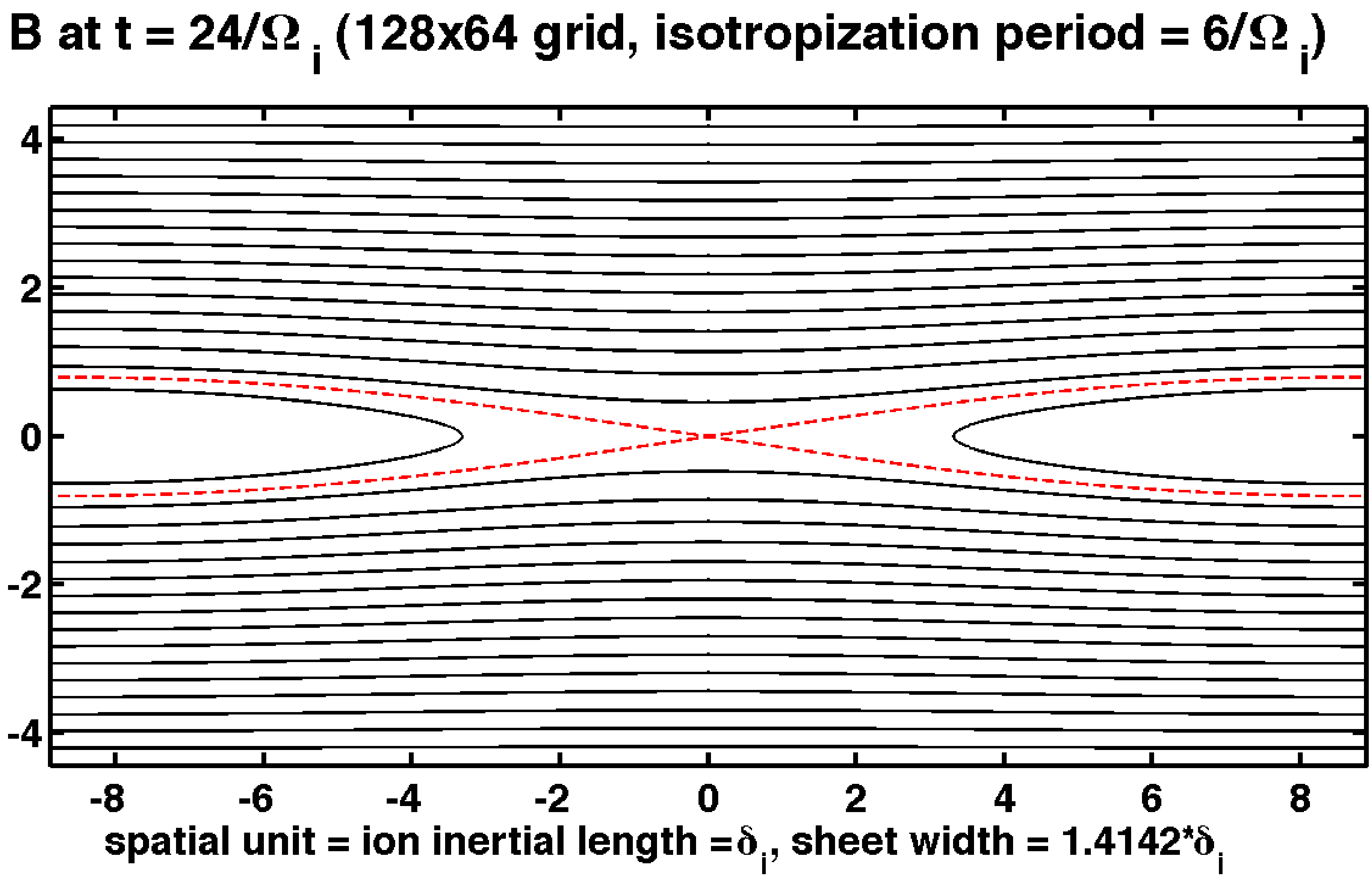
B at t = 18/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



B at t = 20/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

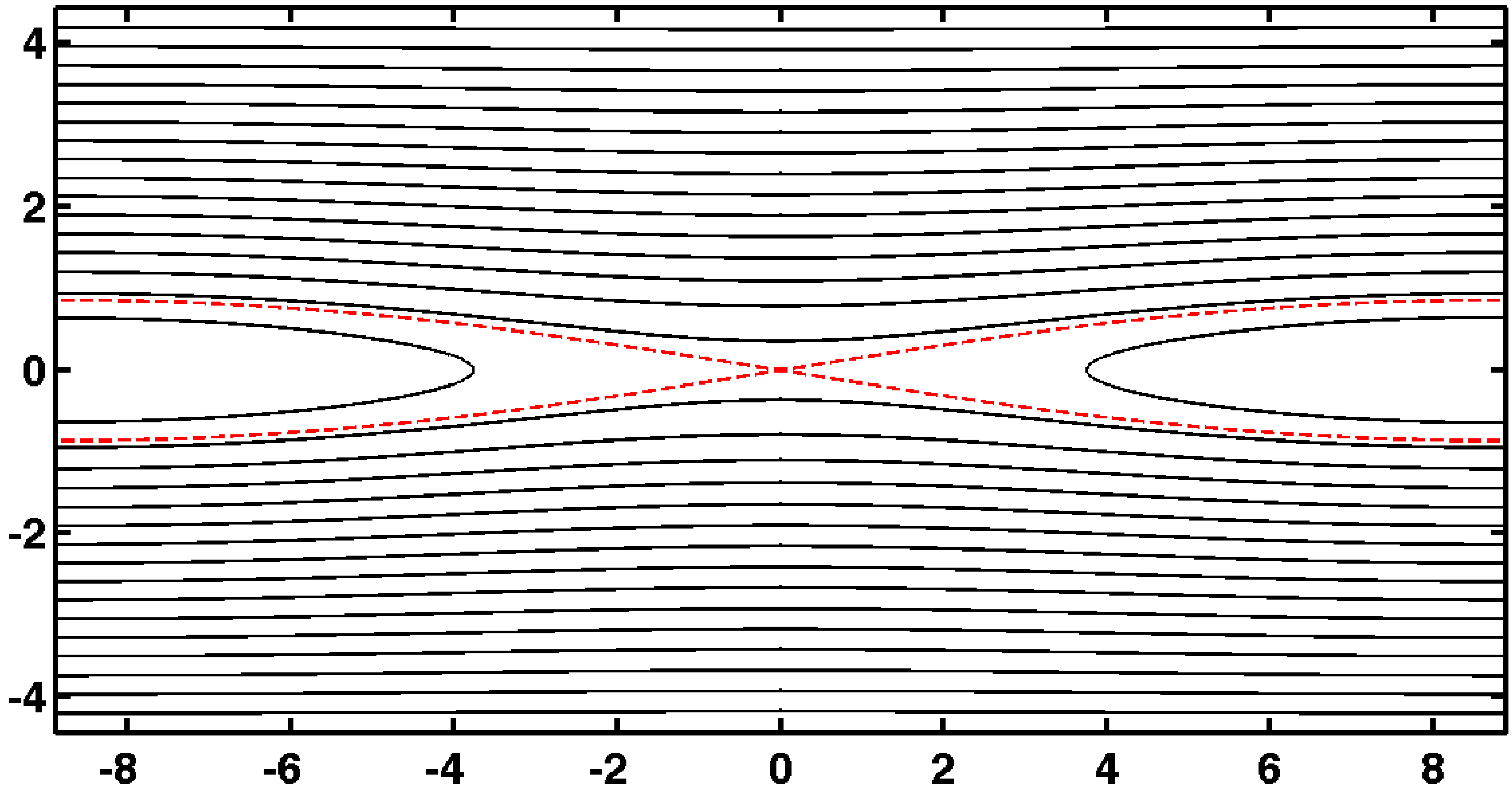


spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i



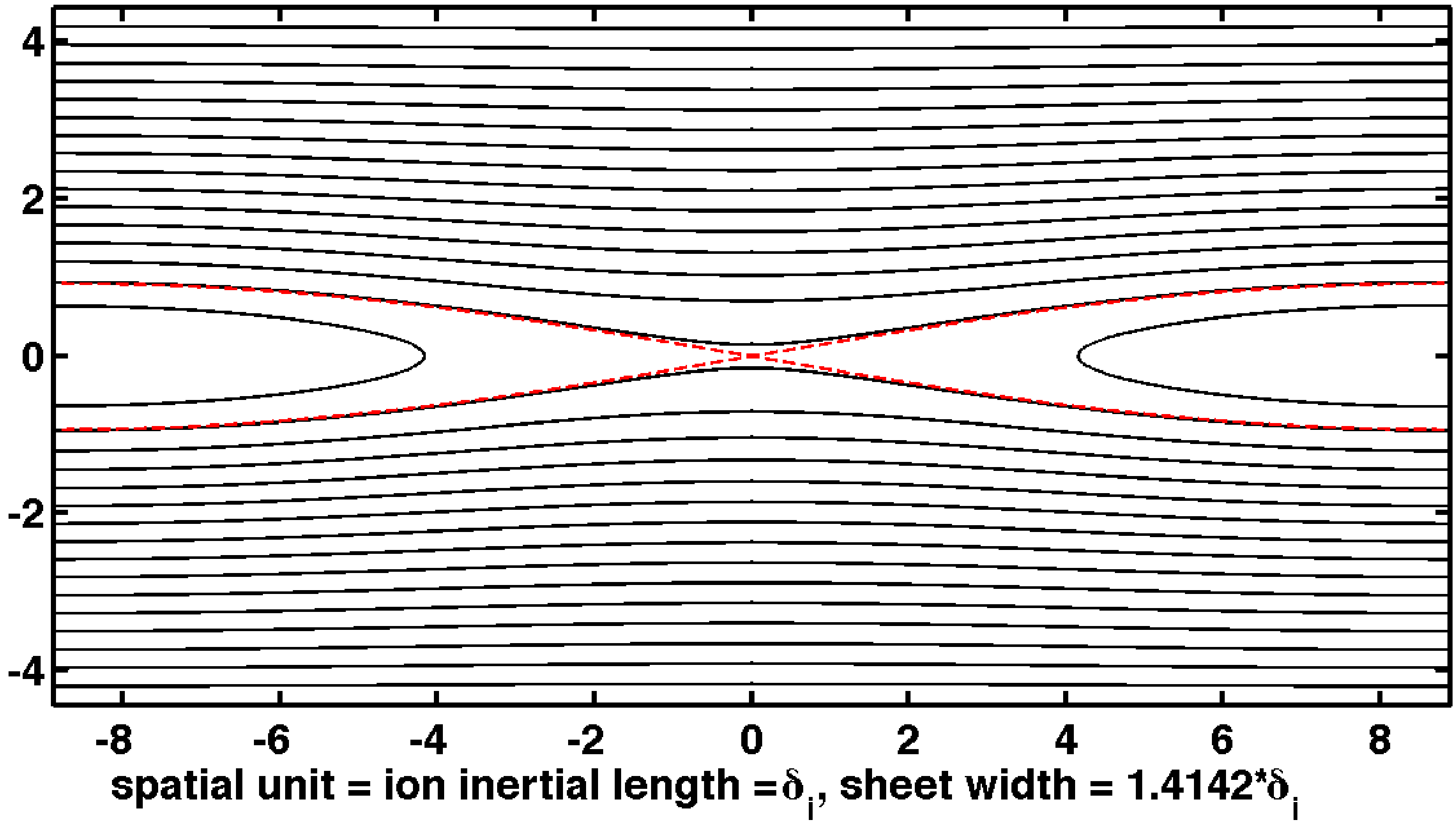
B at t = 24/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

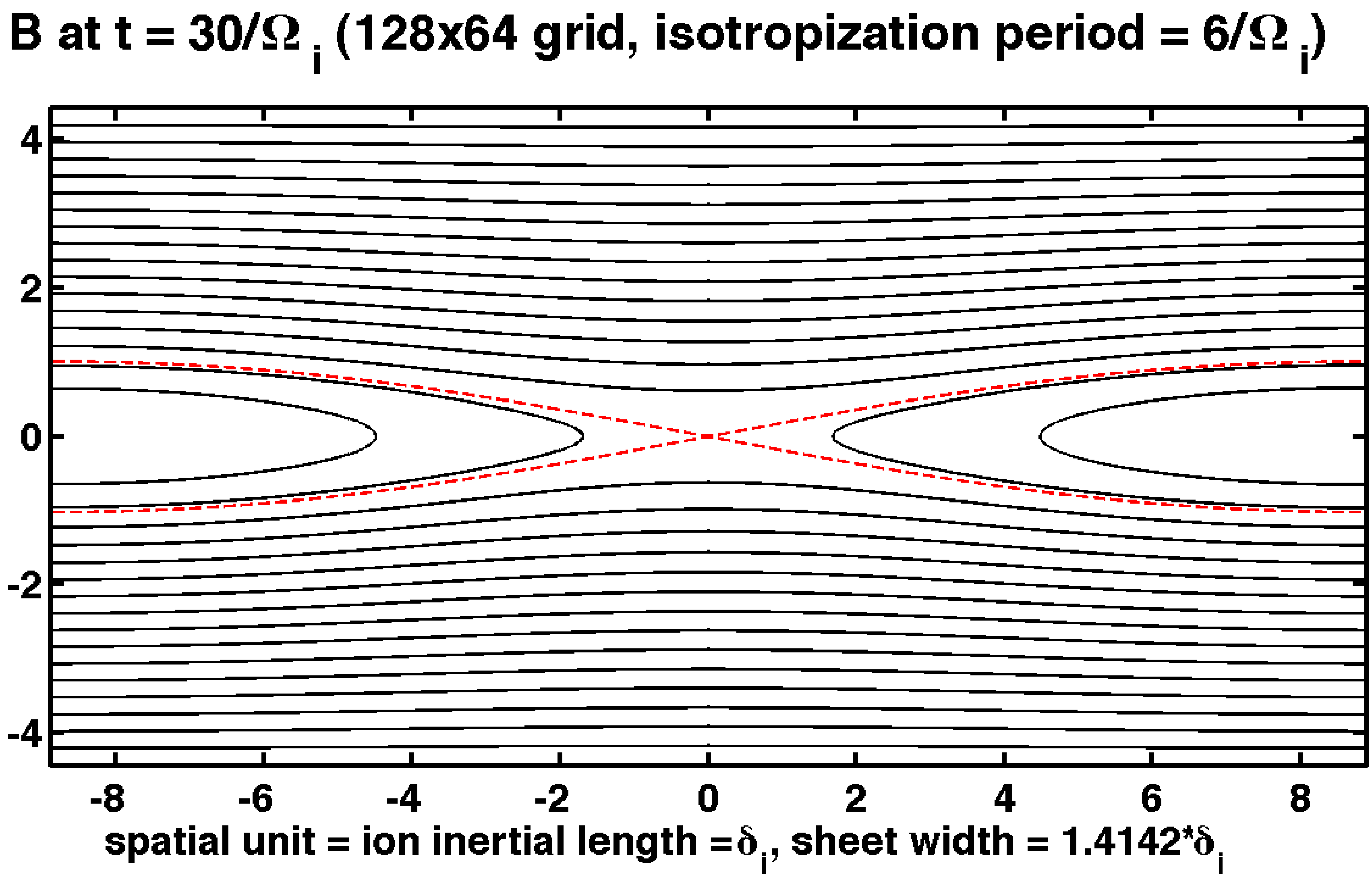
B at t = 26/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

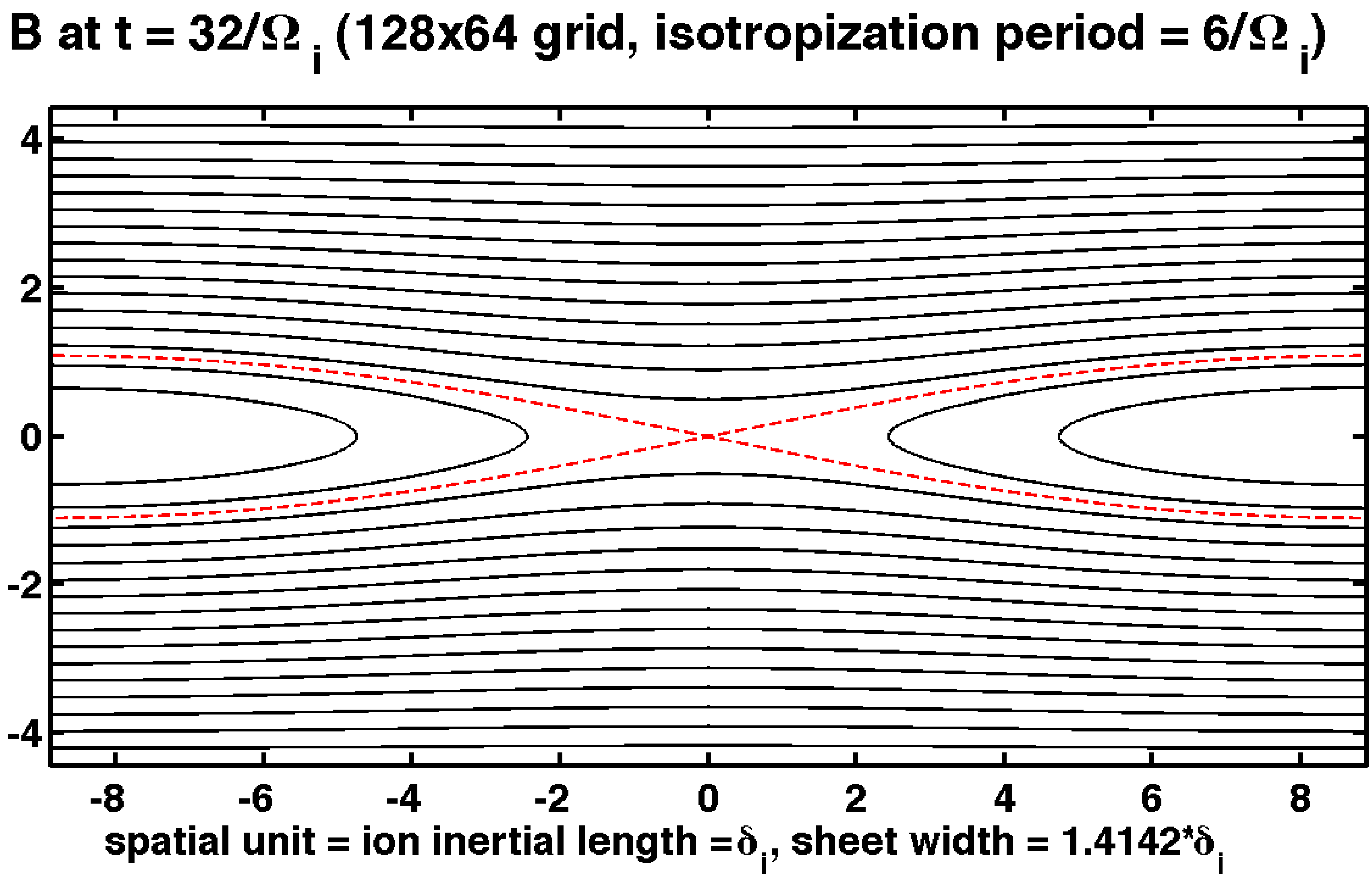


spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

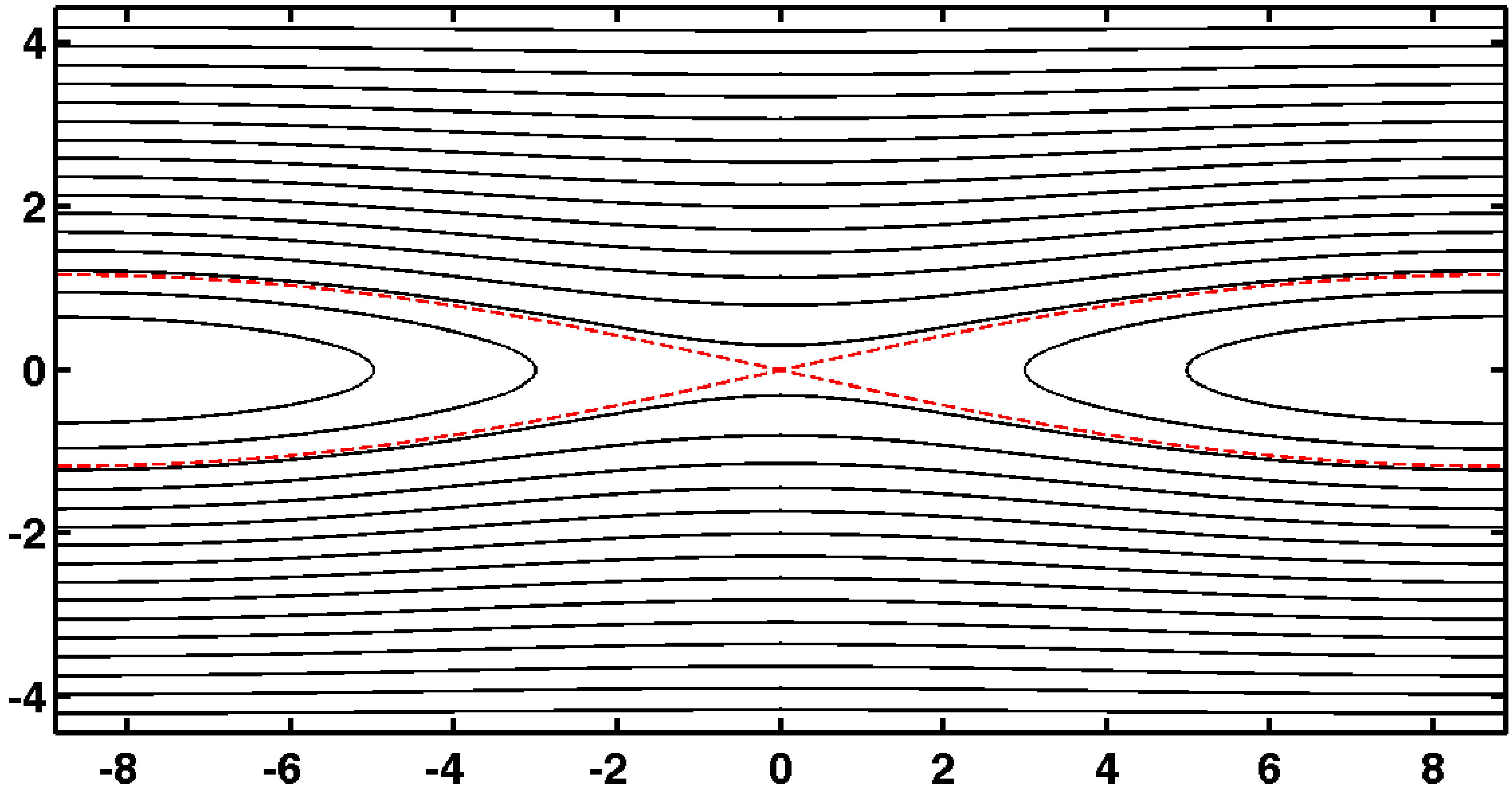
B at t = 28/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



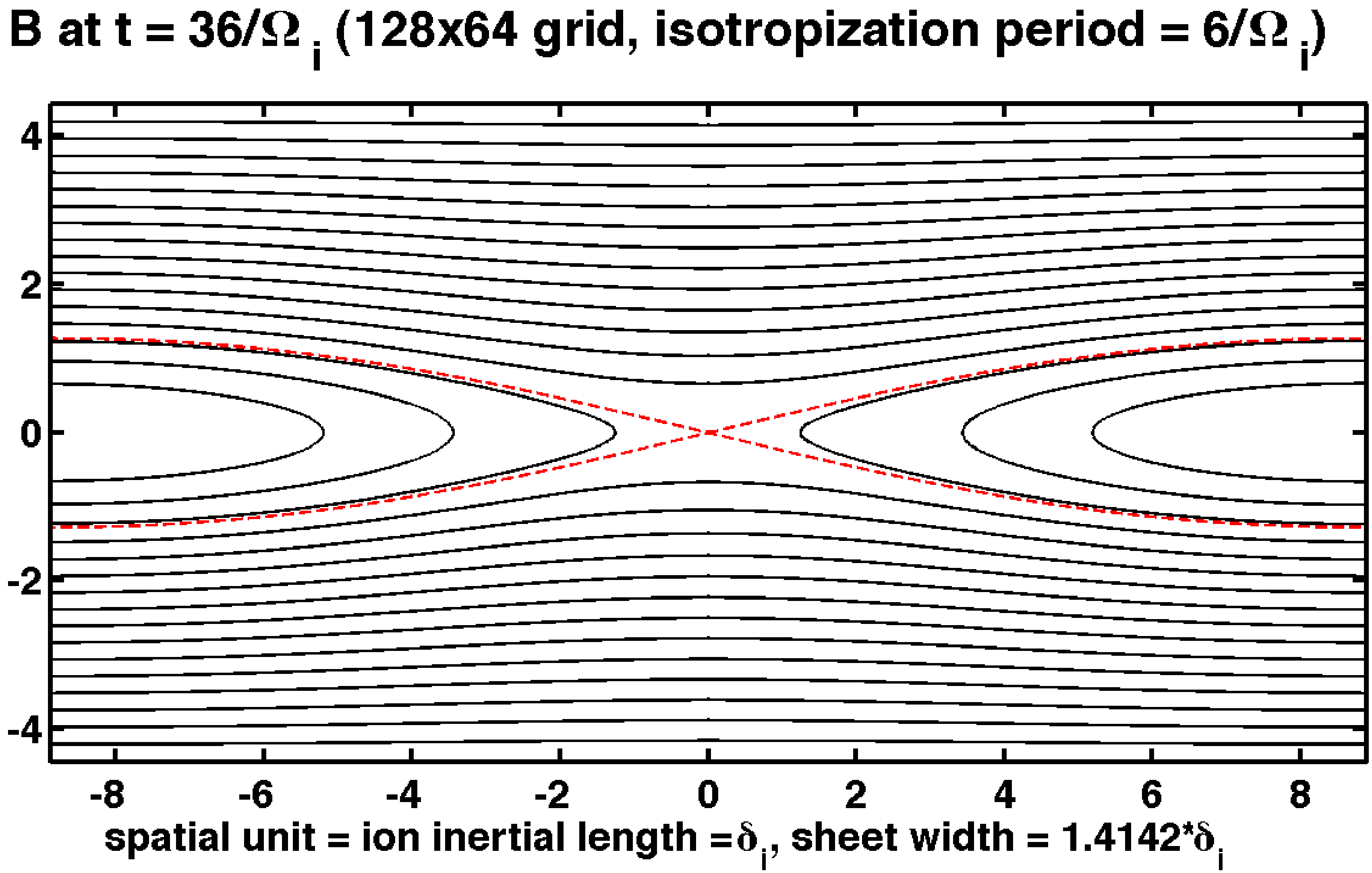

B at t = 30/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 32/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

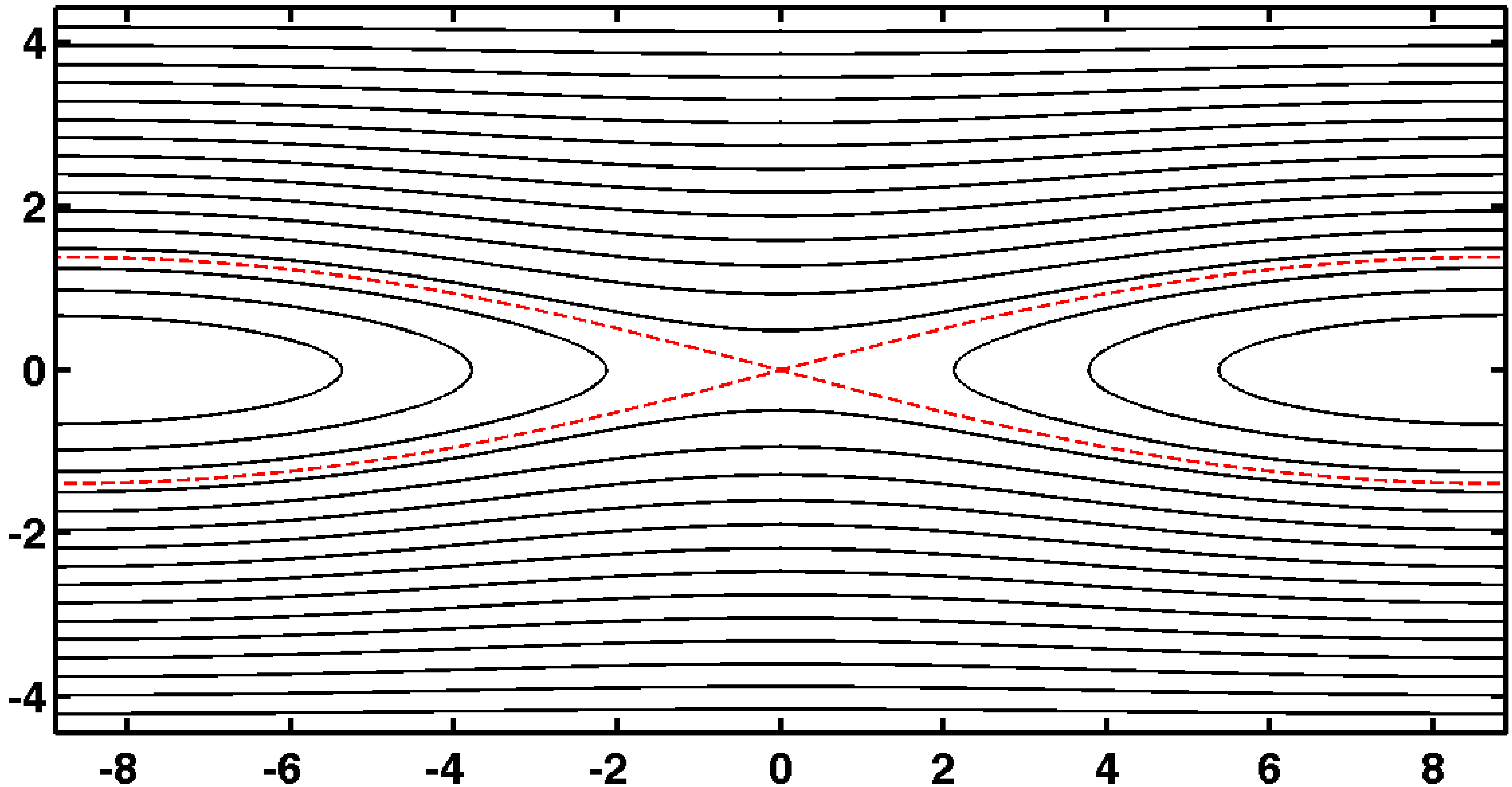
B at t = 34/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



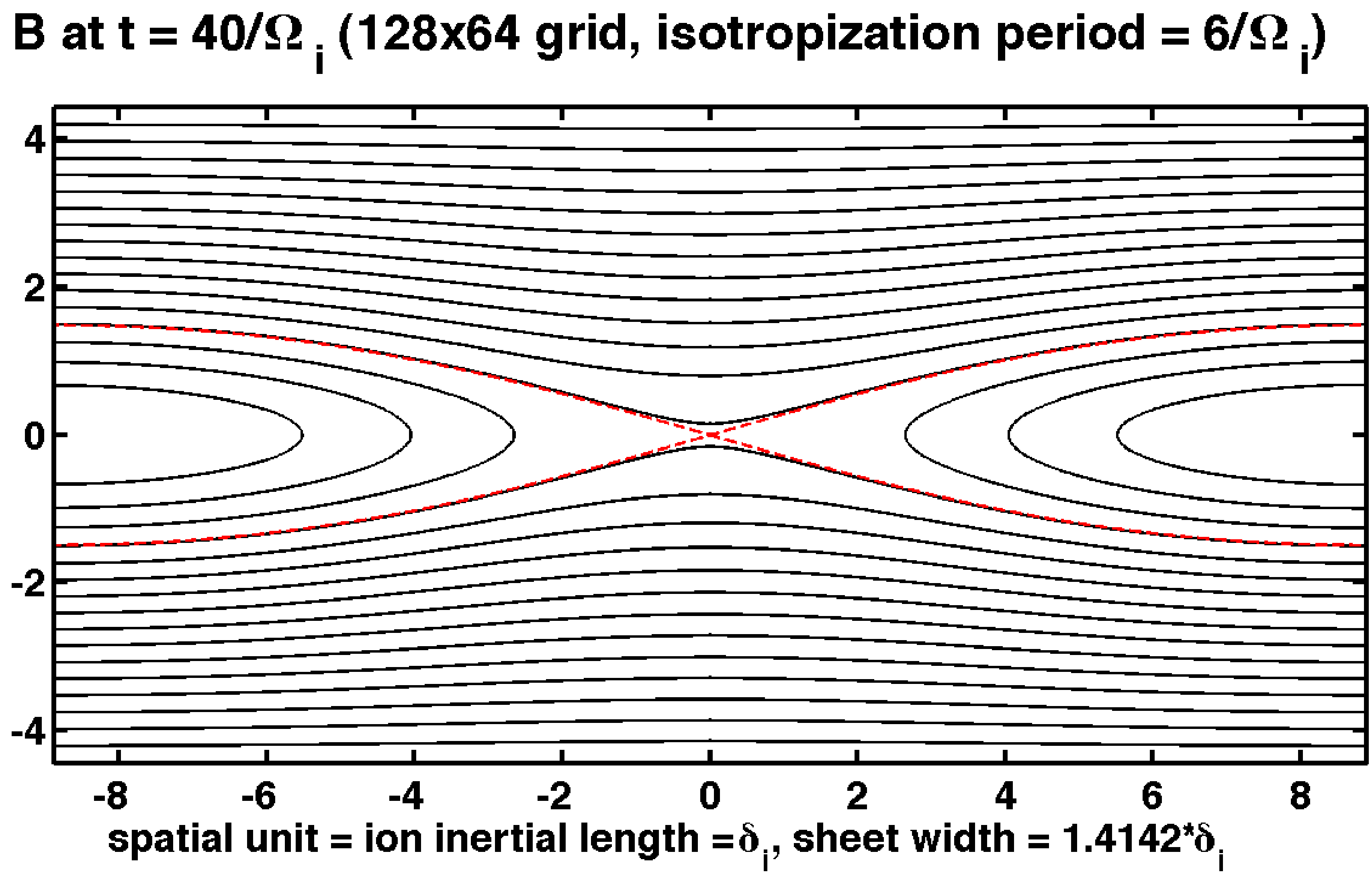
spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i


B at t = 36/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

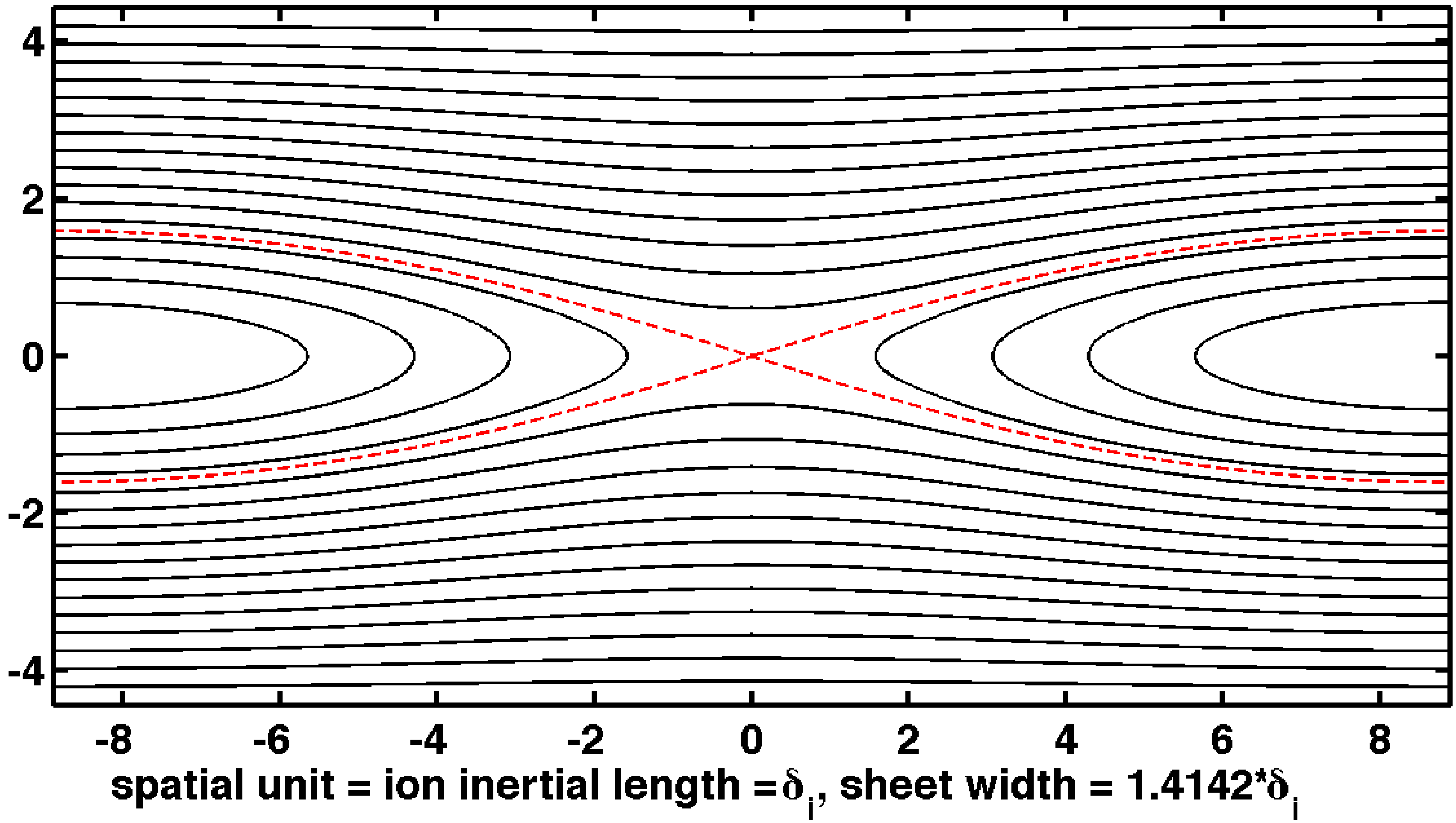
B at t = 38/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

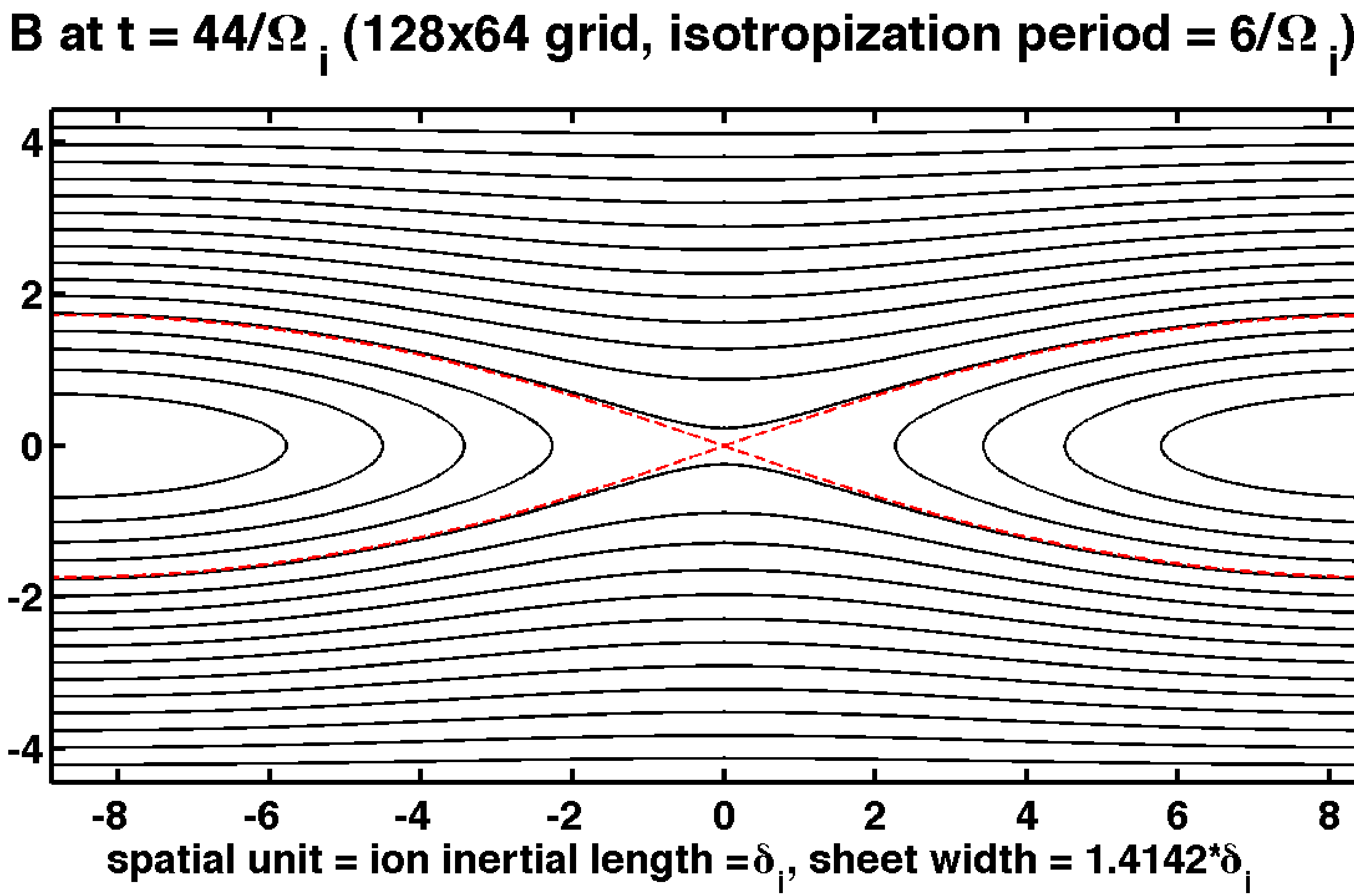


spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

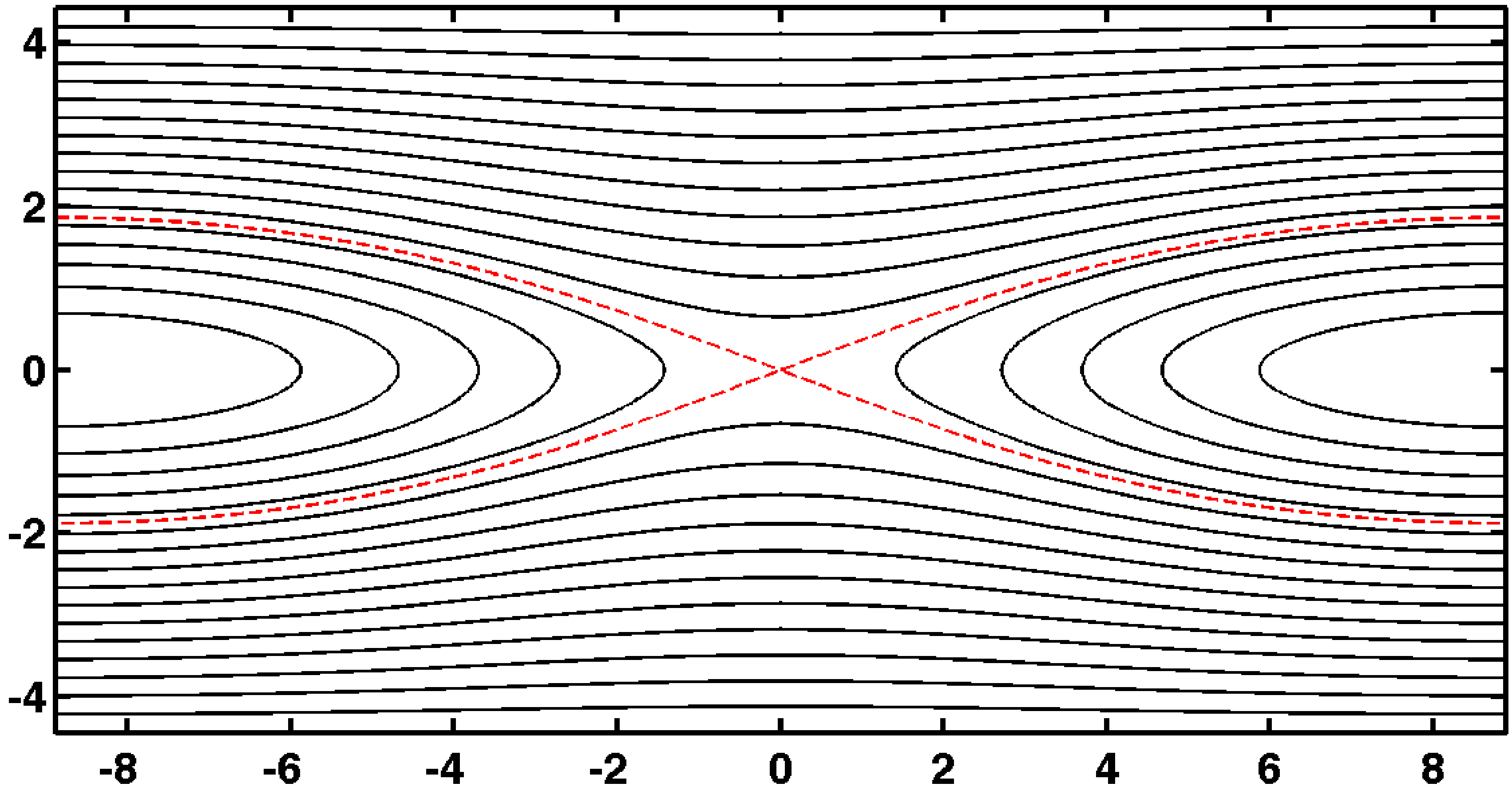

B at t = 40/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

B at t = $42/\Omega_i$ (128x64 grid, isotropization period = $6/\Omega_i$)



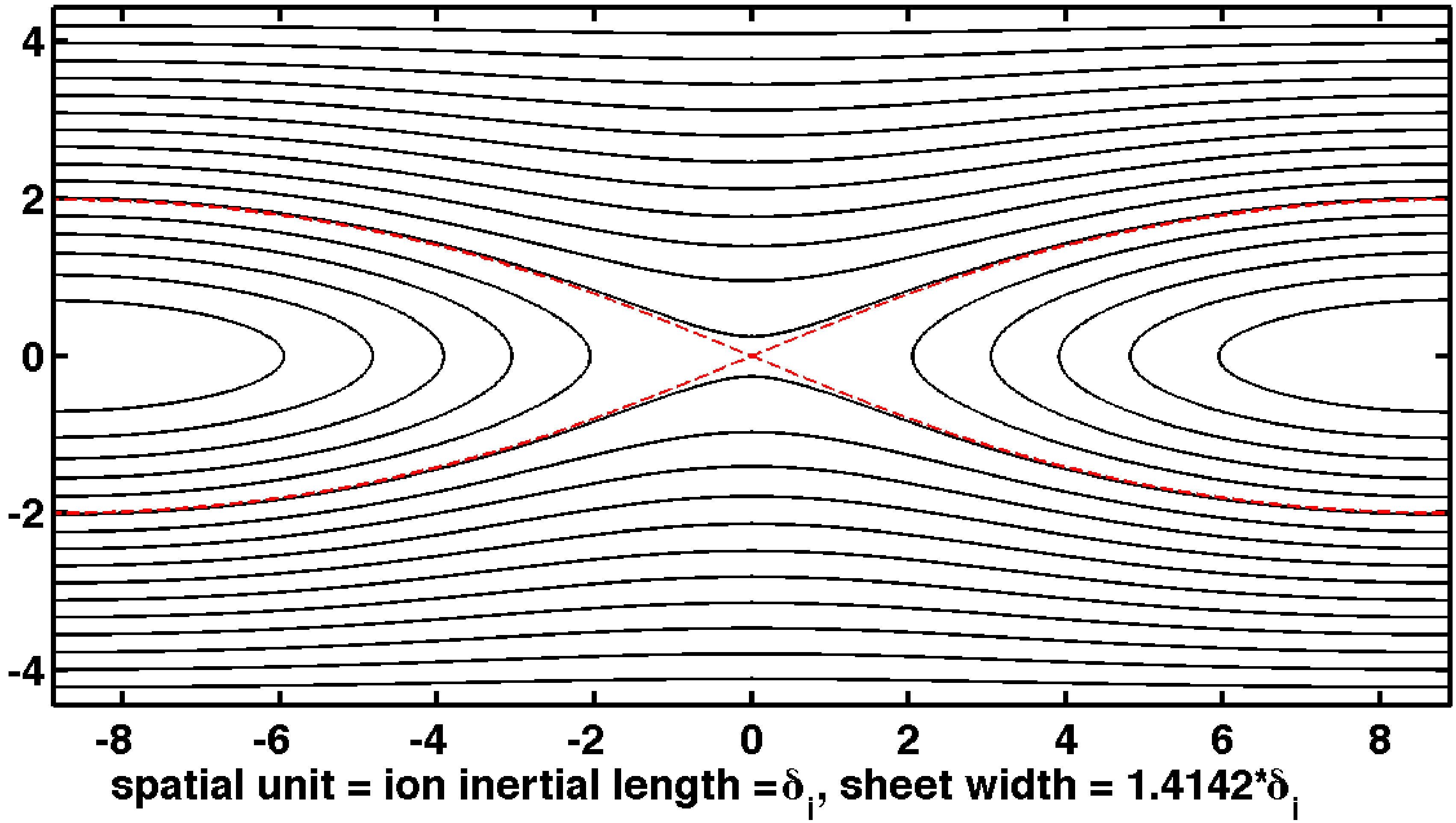

B at t = 44/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

B at t = 46/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

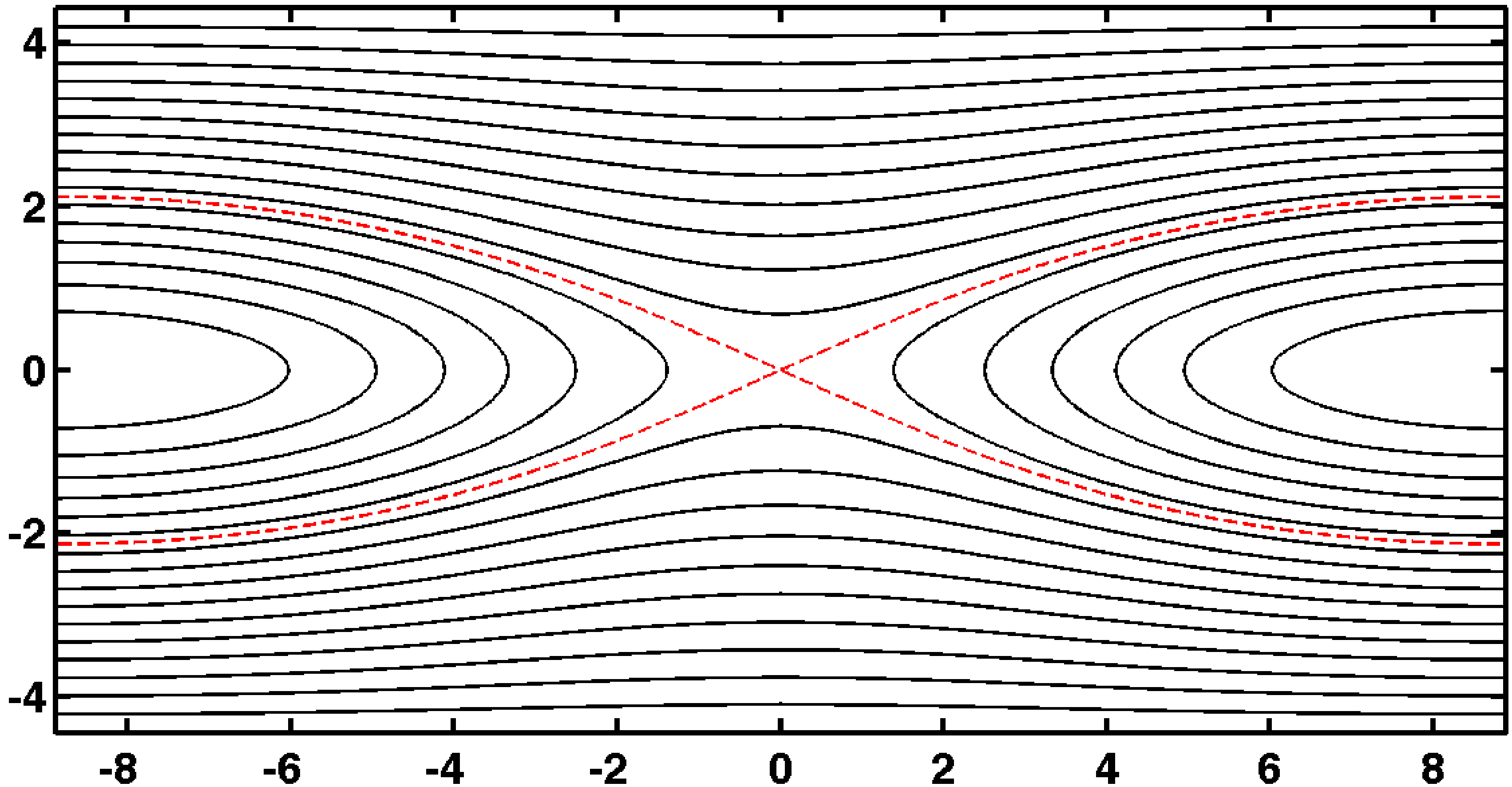


spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

B at t = 48/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

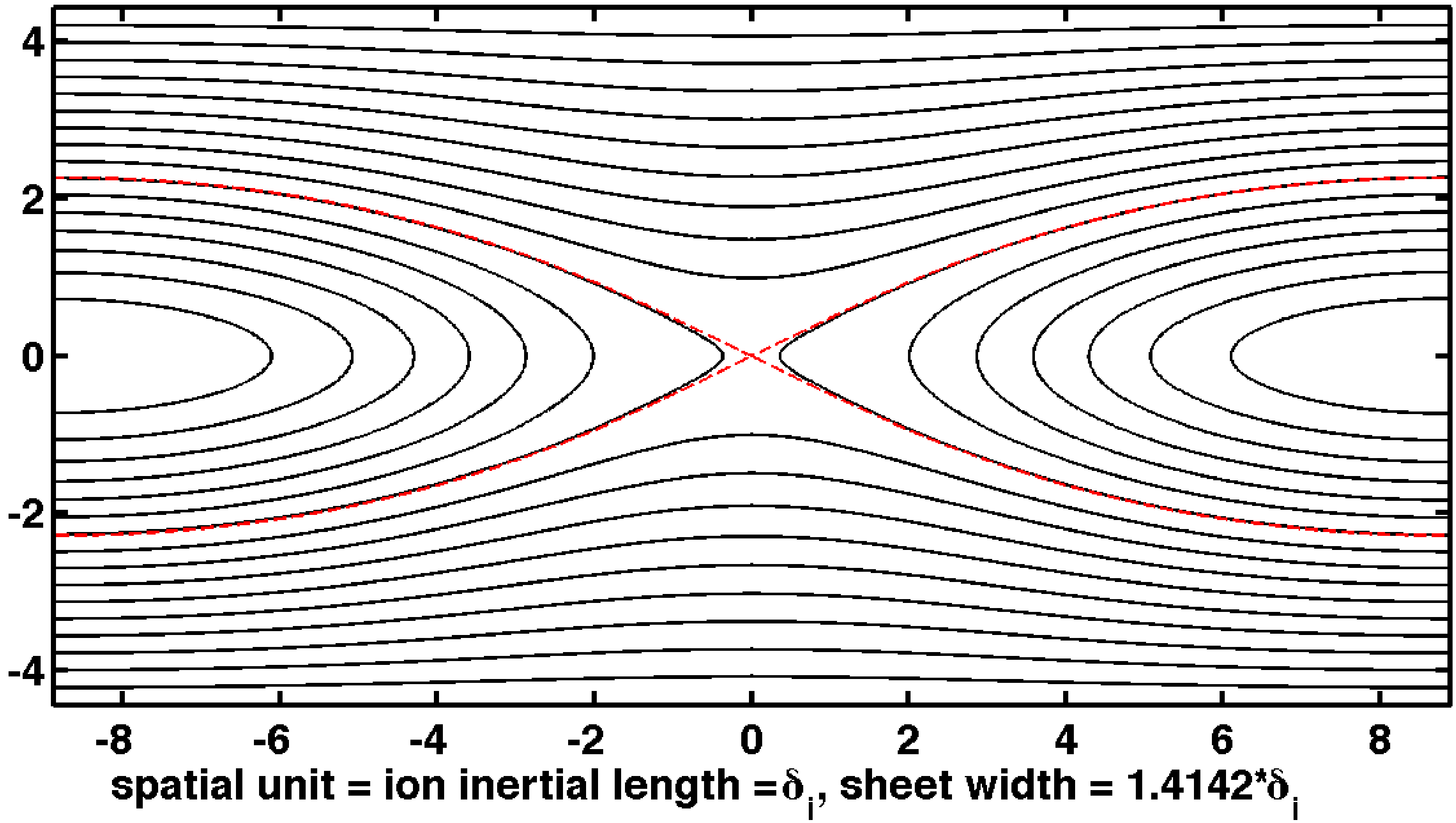


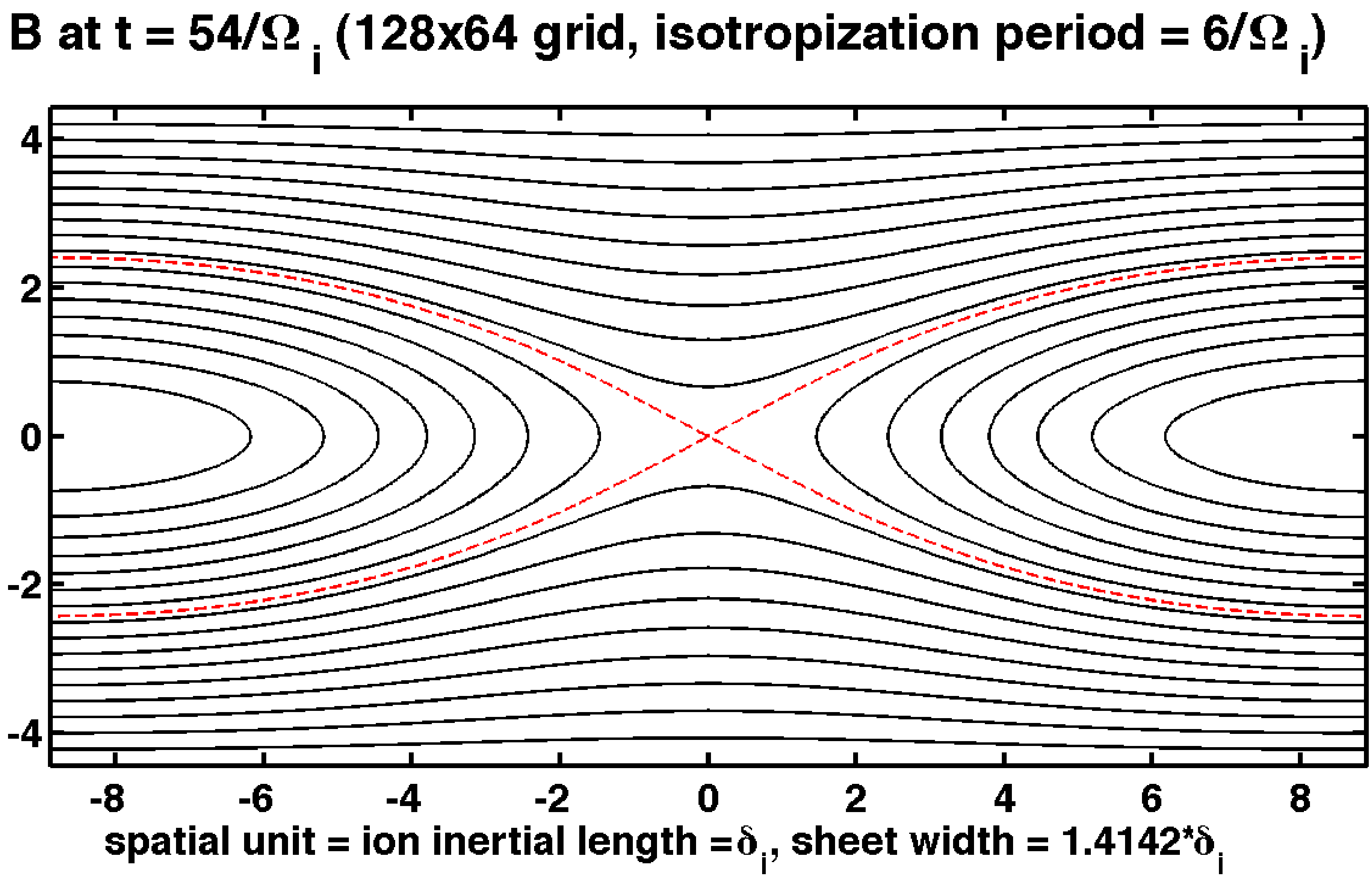
B at t = 50/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



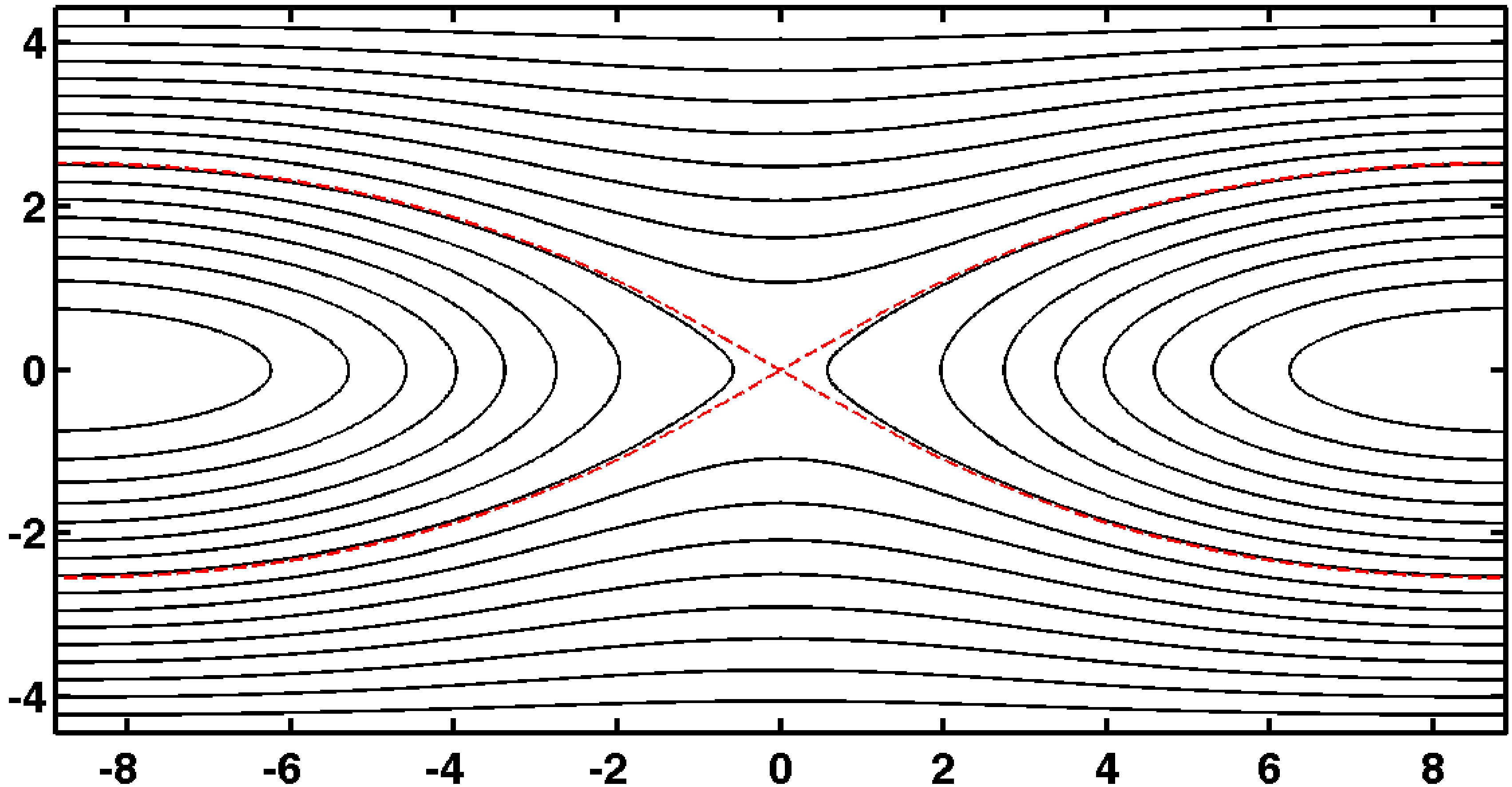
spatial unit = ion inertial length = δ_i, sheet width = 1.4142*δ_i

B at t = $52/\Omega_i$ (128x64 grid, isotropization period = $6/\Omega_i$)

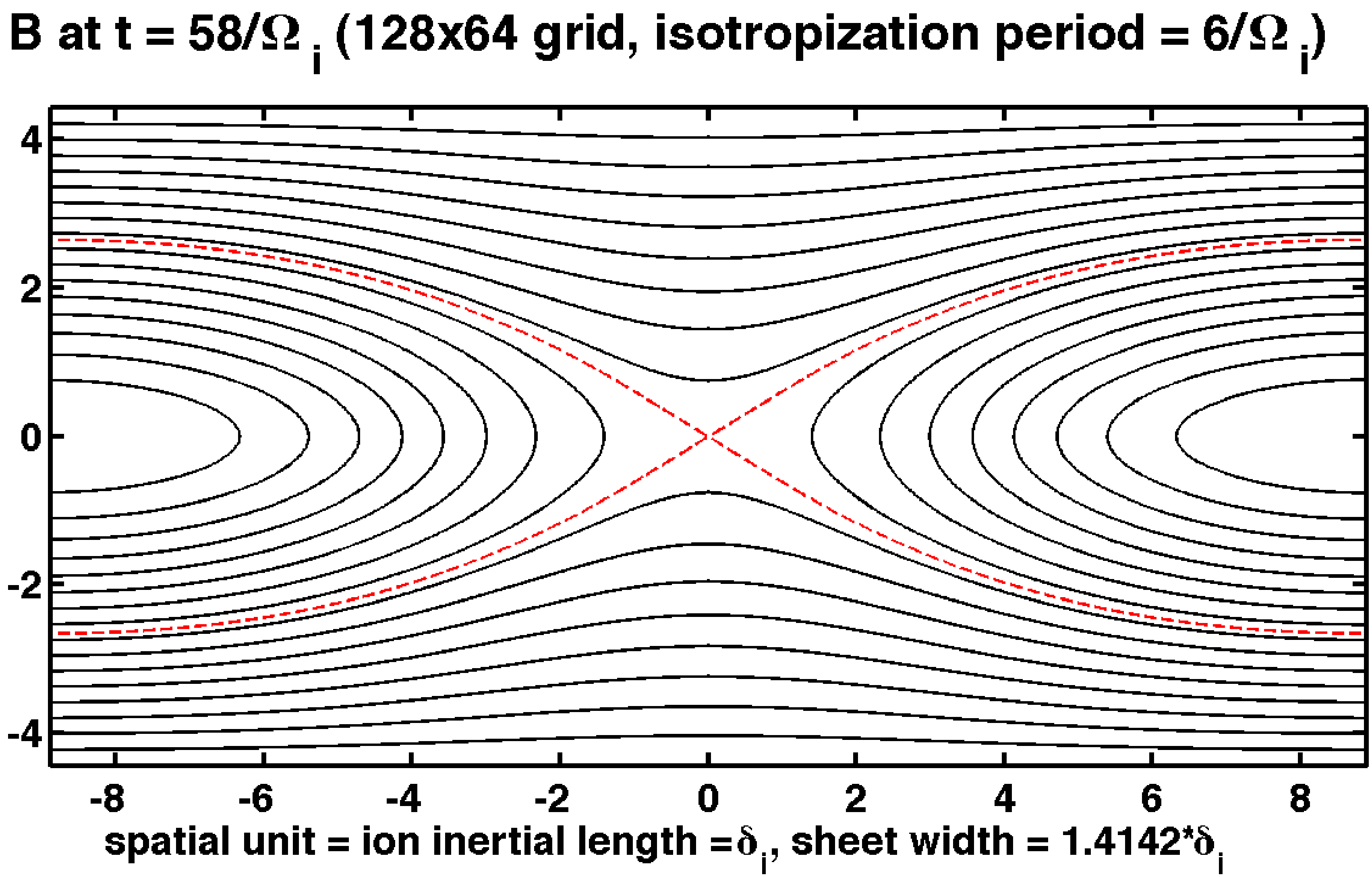


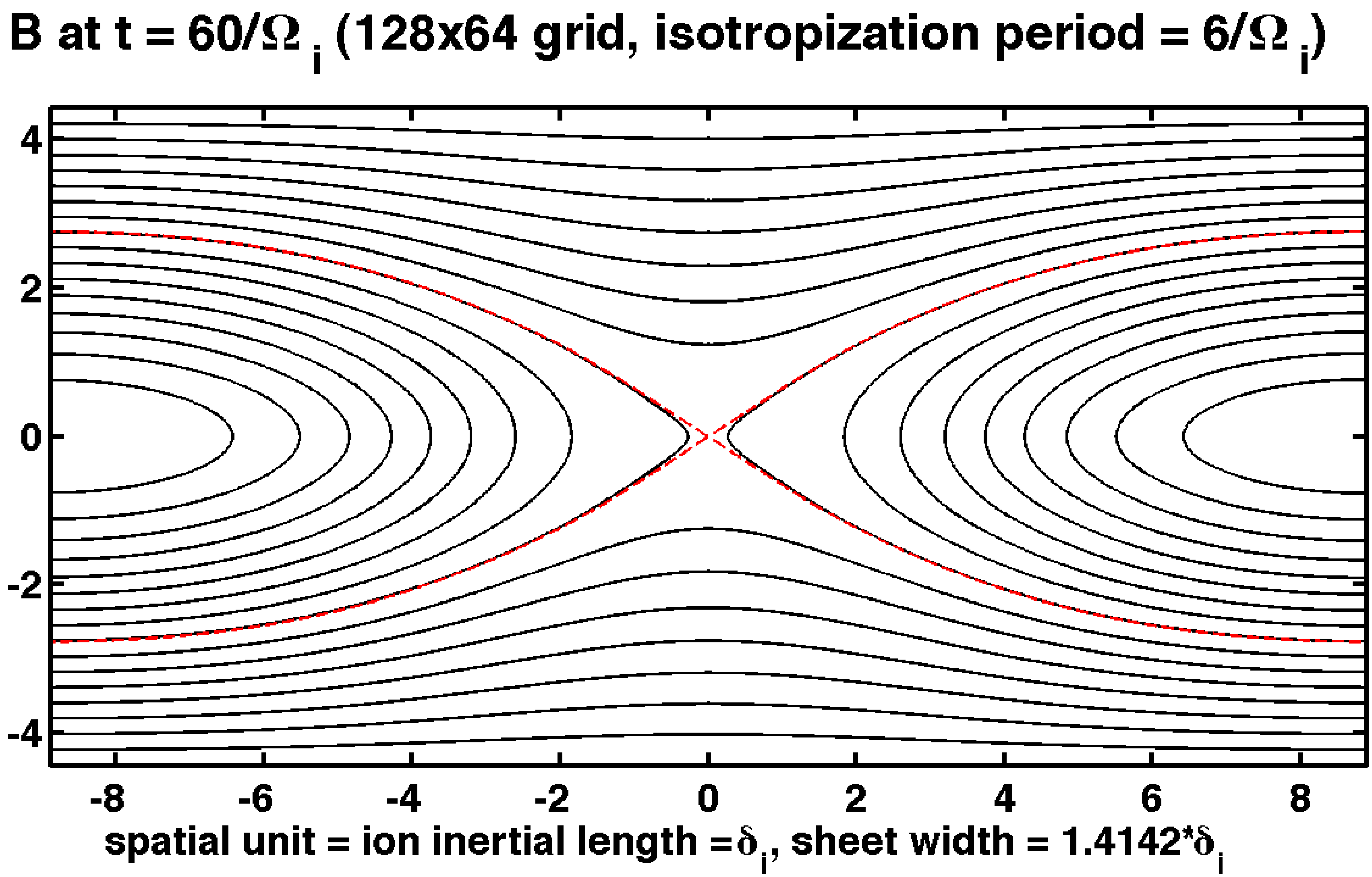

B at t = 54/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

B at t = 56/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

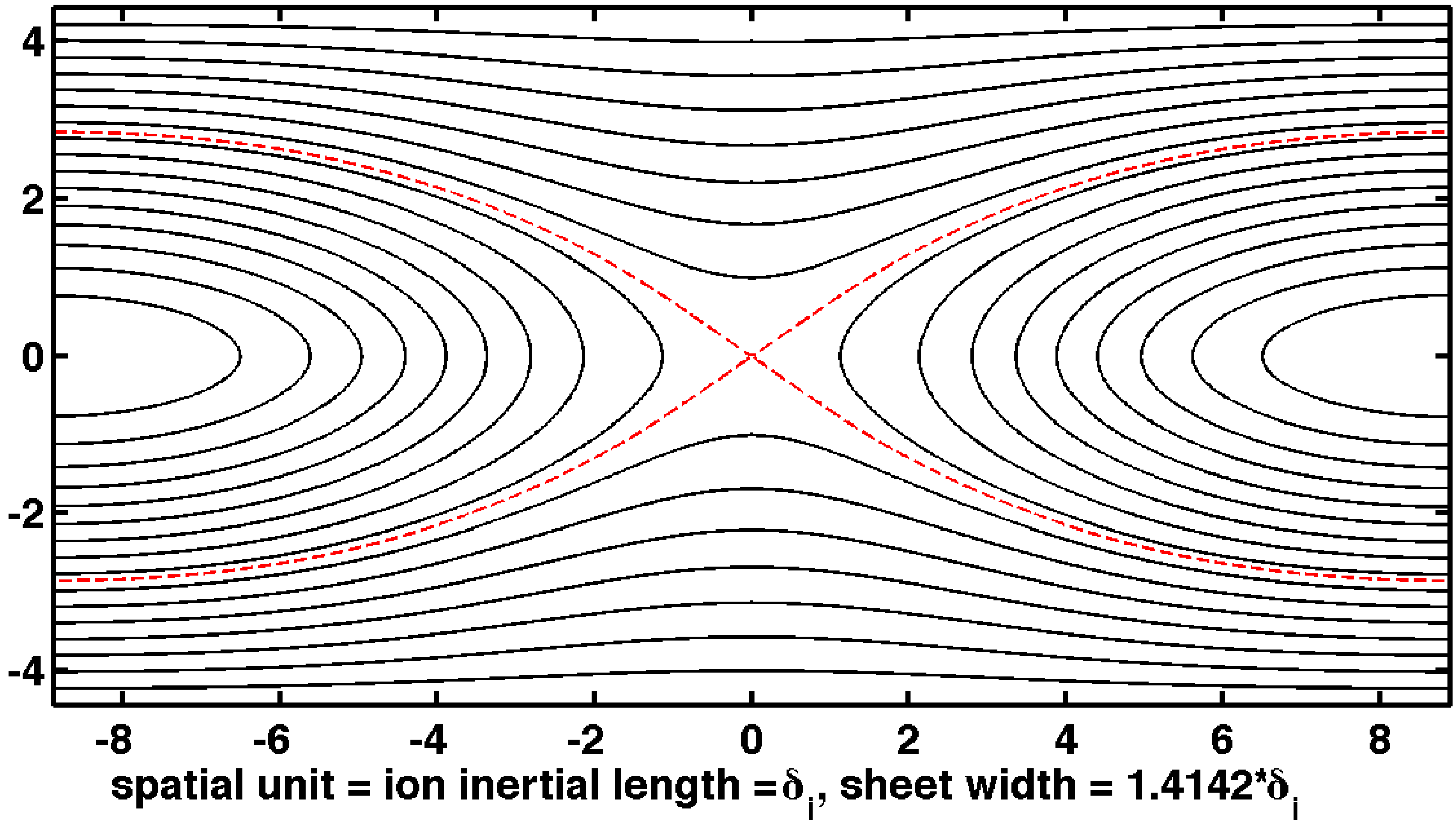


spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i

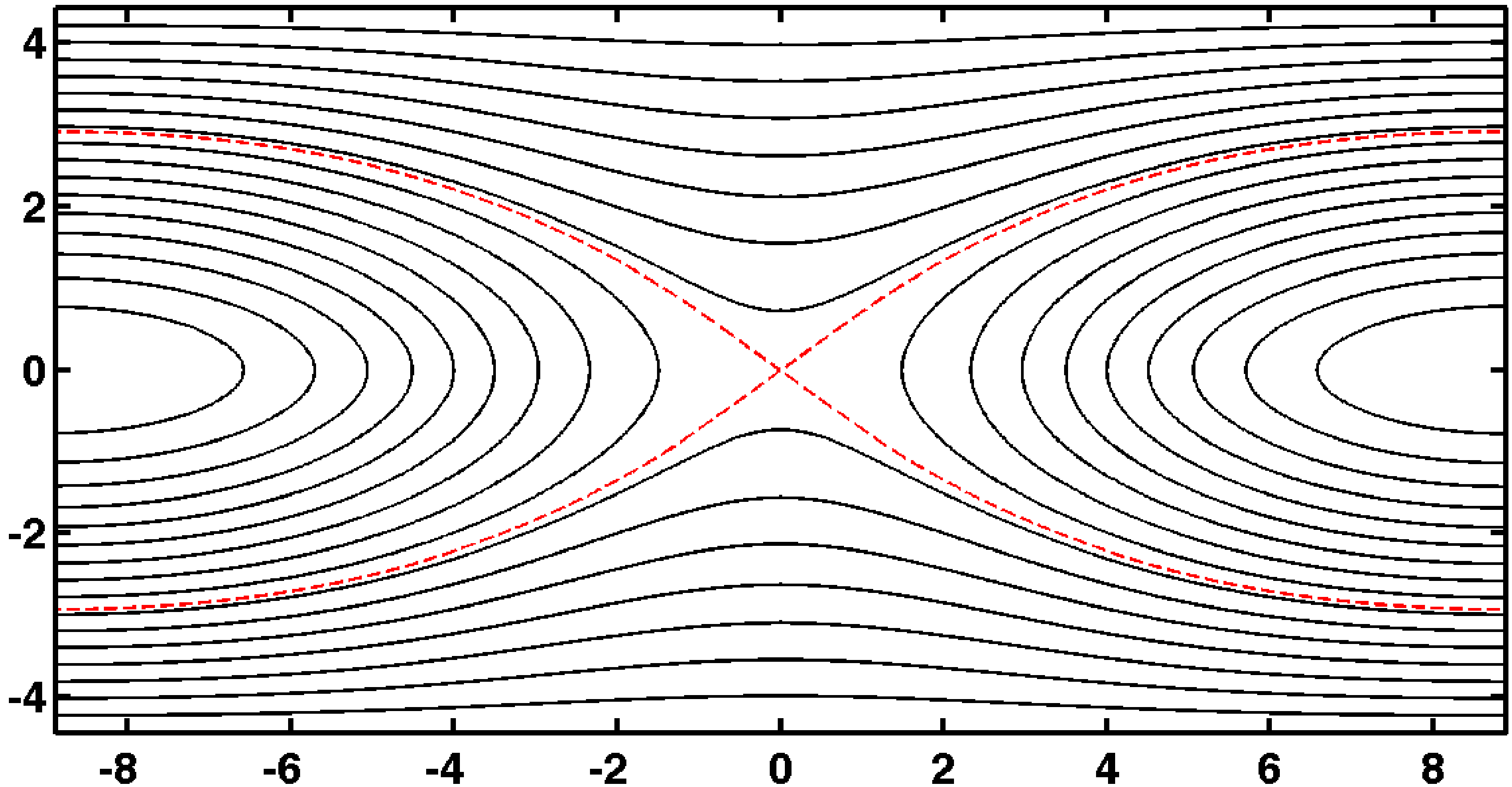

B at t = 58/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 60/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

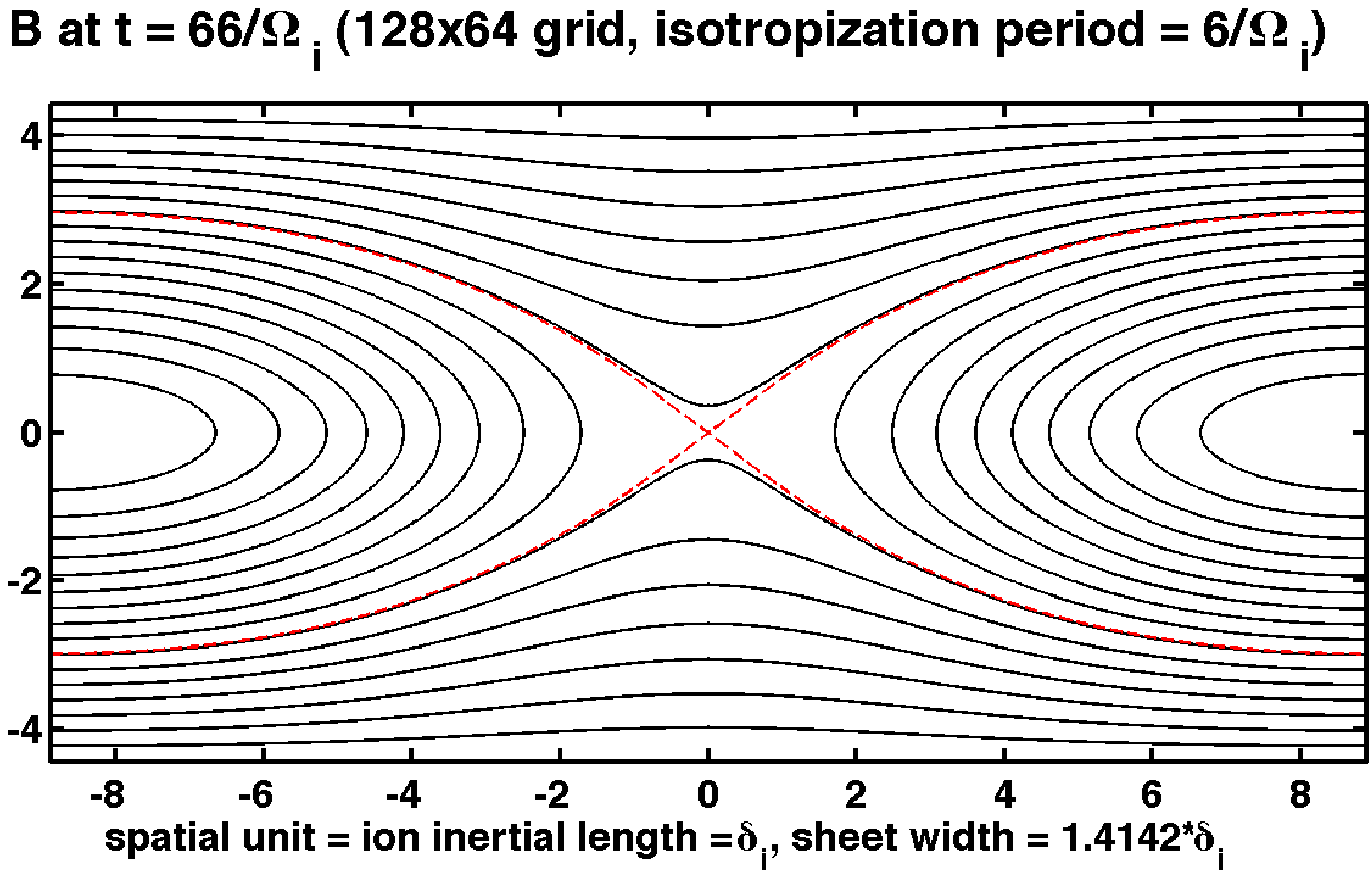
B at t = $62/\Omega_i$ (128x64 grid, isotropization period = $6/\Omega_i$)

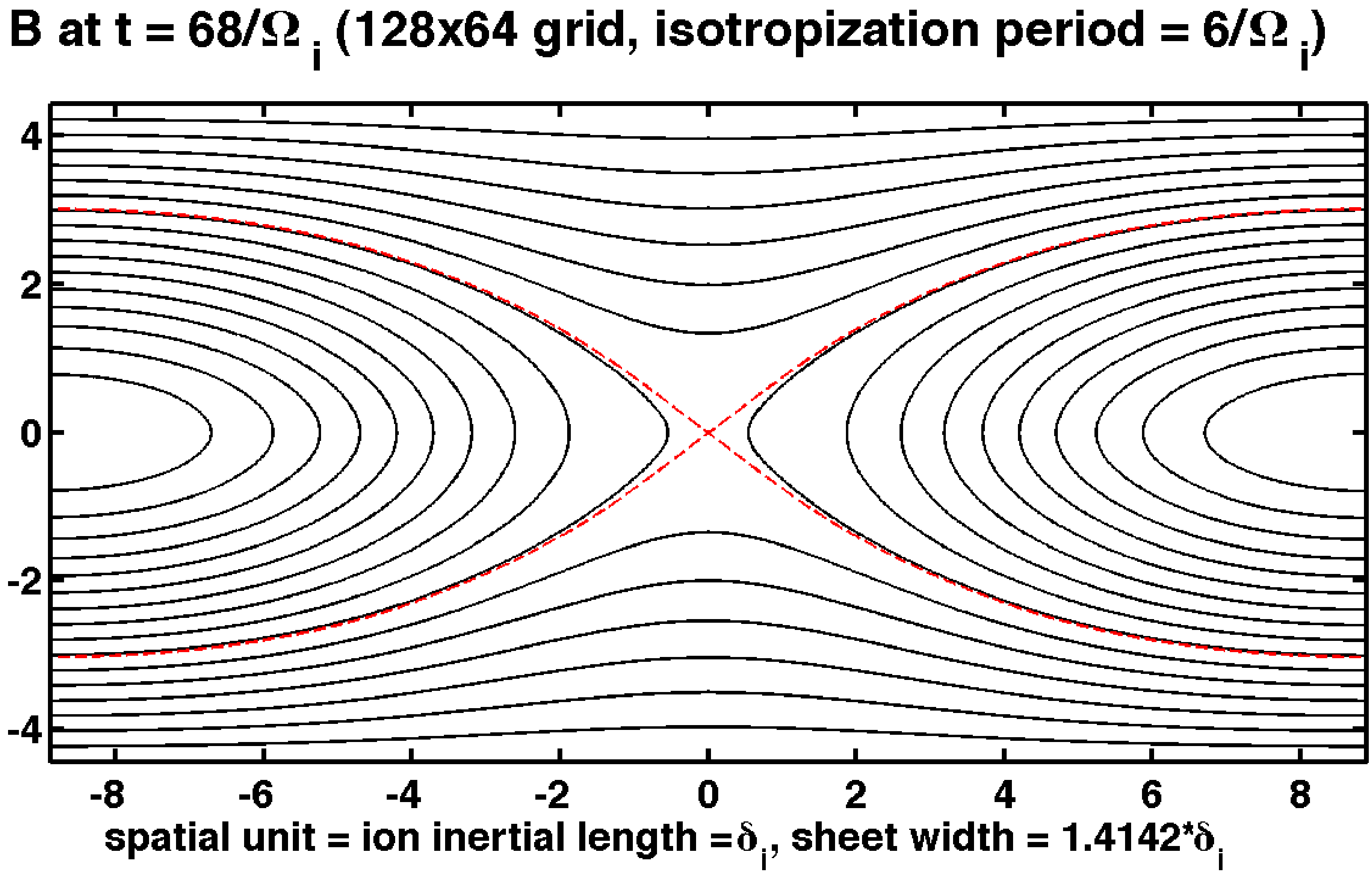


B at t = 64/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

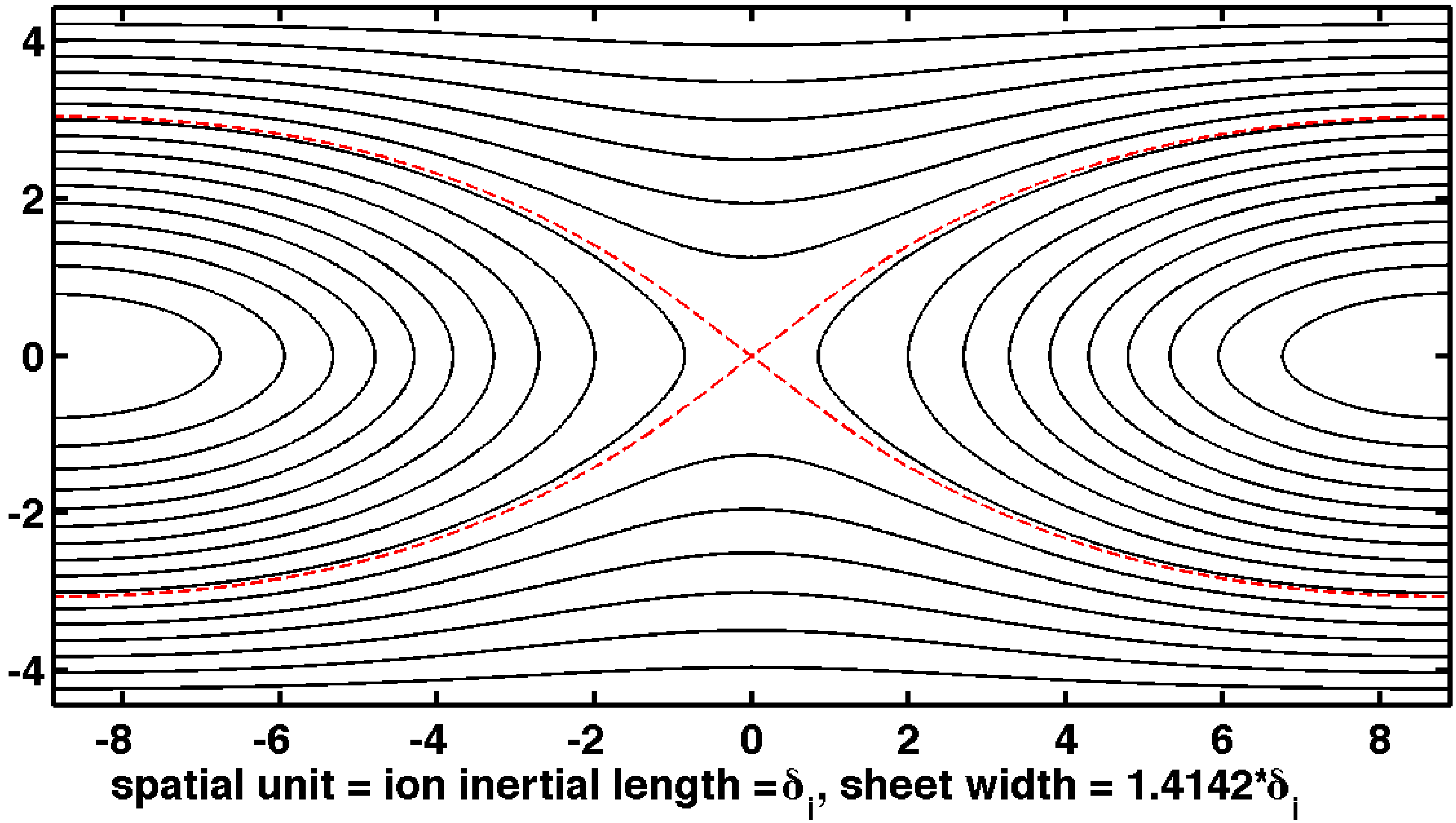


spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i

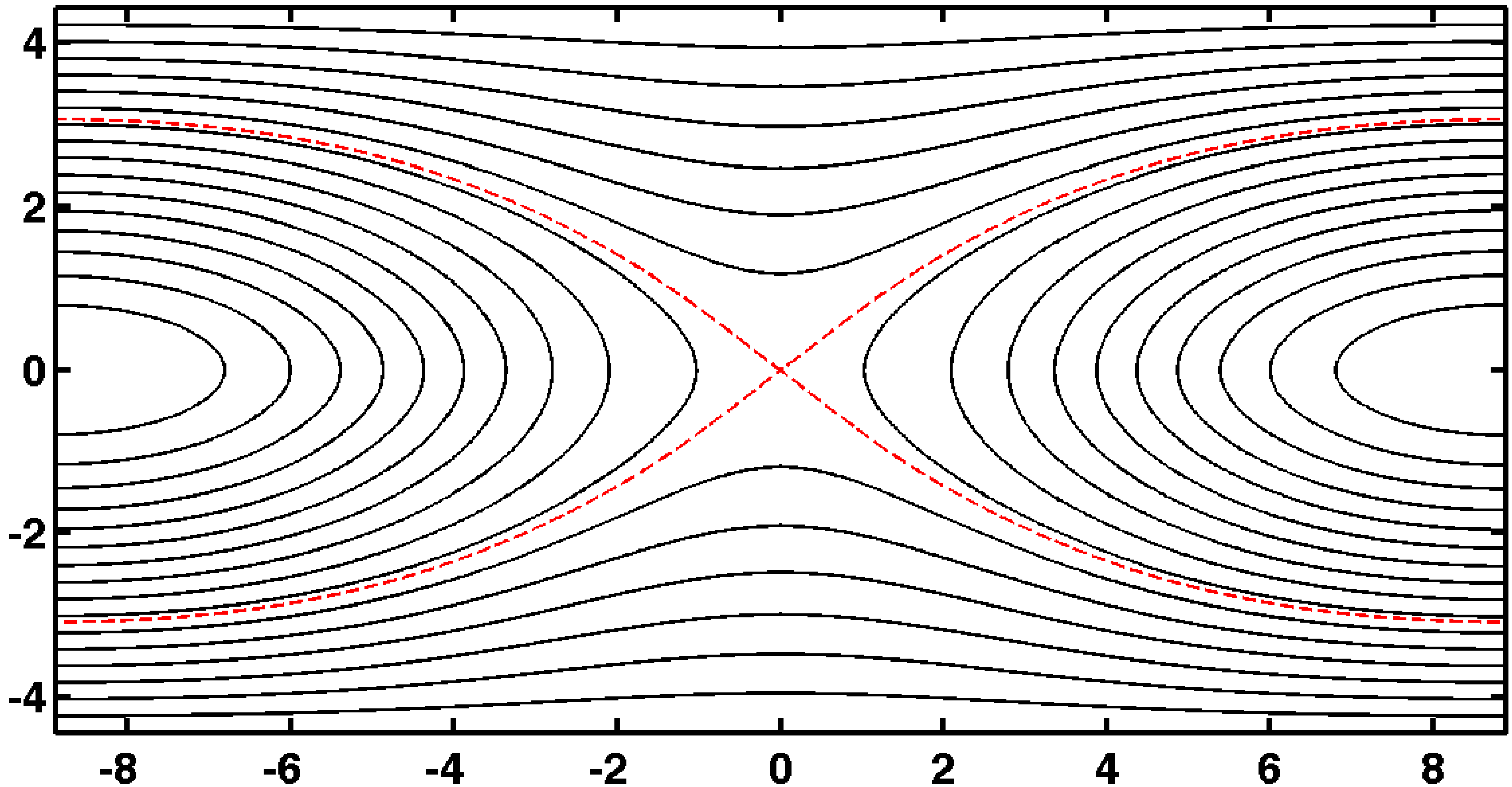

B at t = 66/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 68/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

B at t = 70/Ω$_i$ (128x64 grid, isotropization period = 6/Ω$_i$)

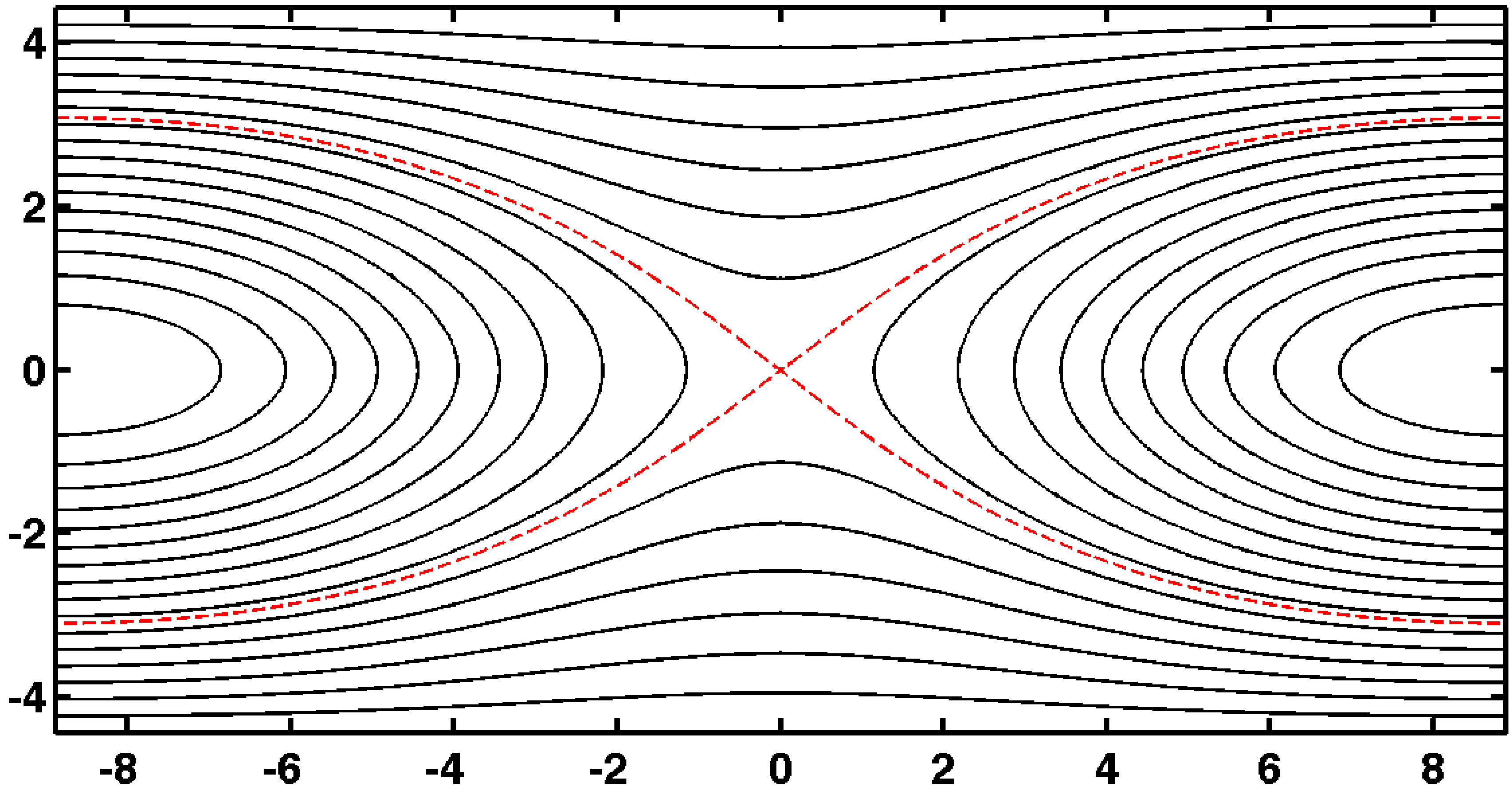


B at t = 72/Ω_i (128x64 grid, isotropization period = 6/Ω_i)

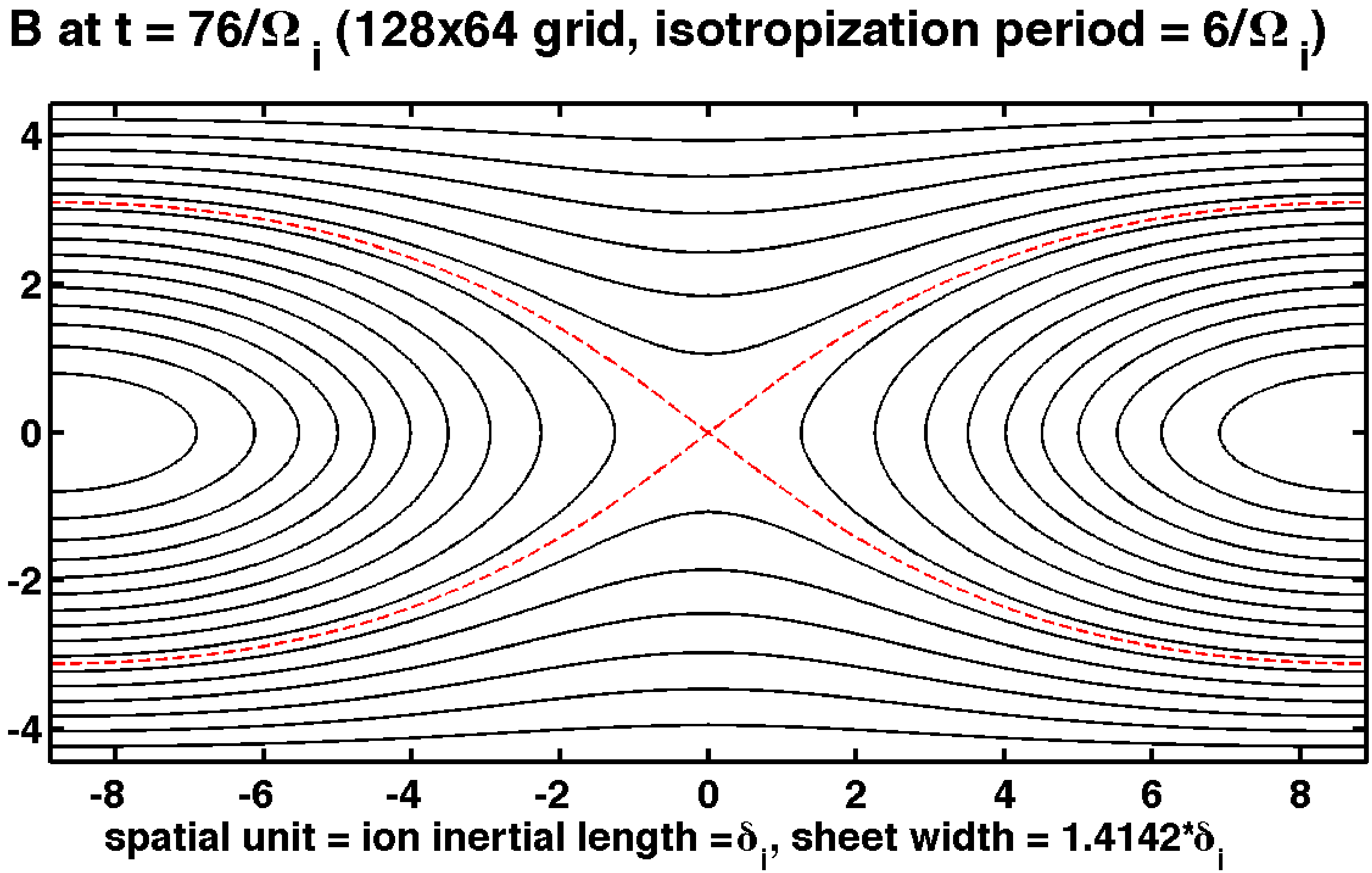


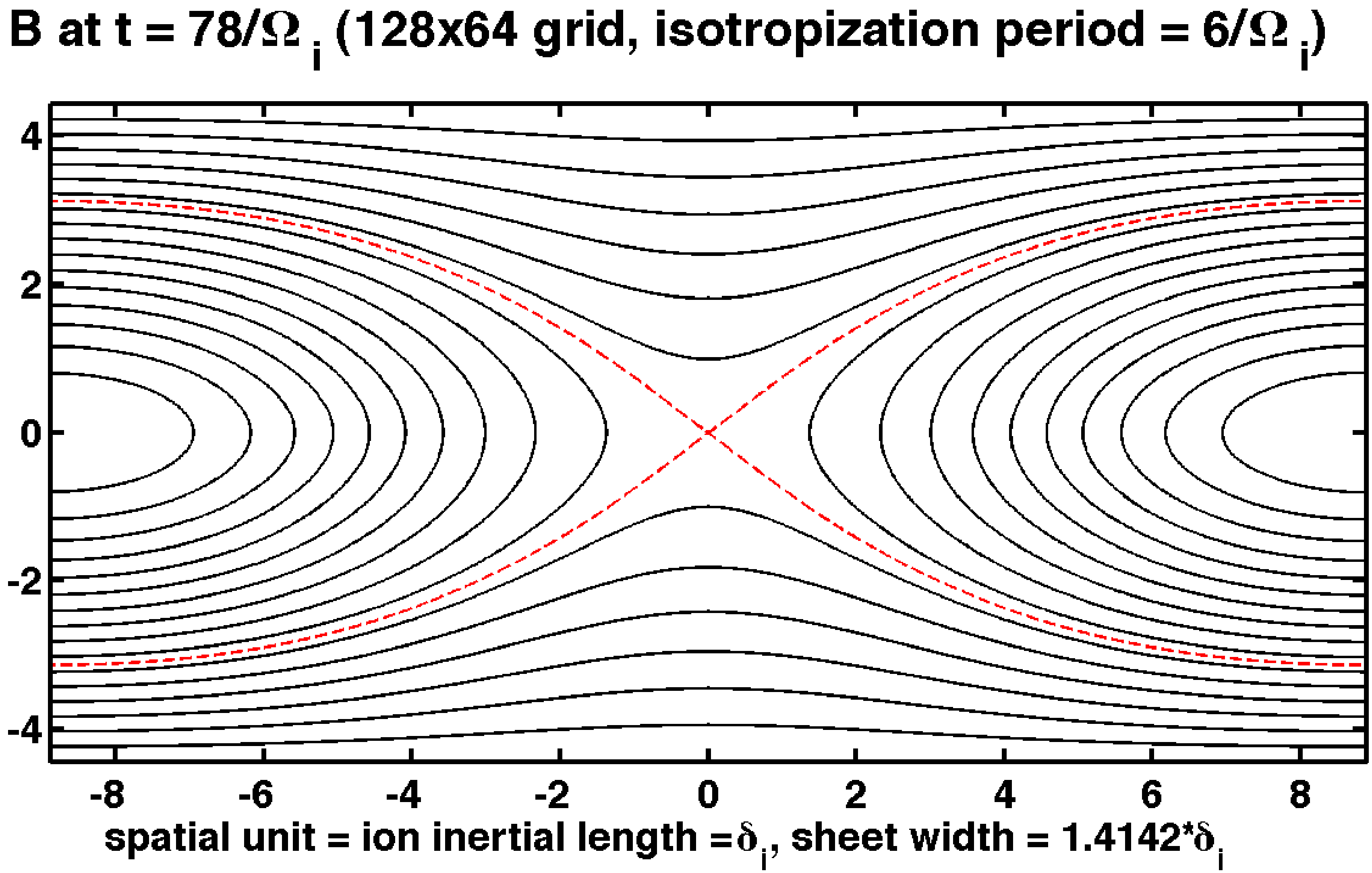
spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i

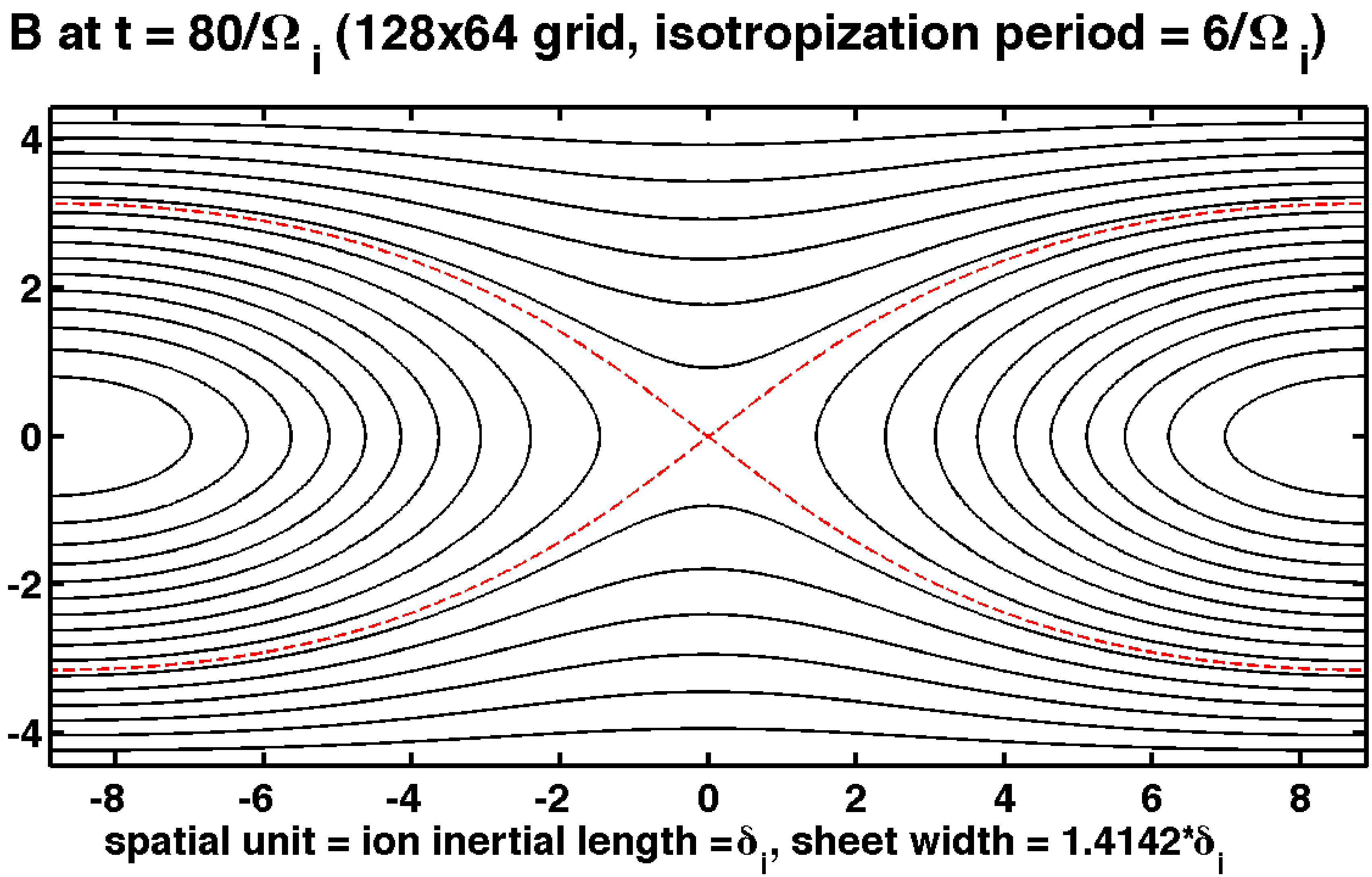
B at t = 74/Ω_i (128x64 grid, isotropization period = 6/Ω_i)



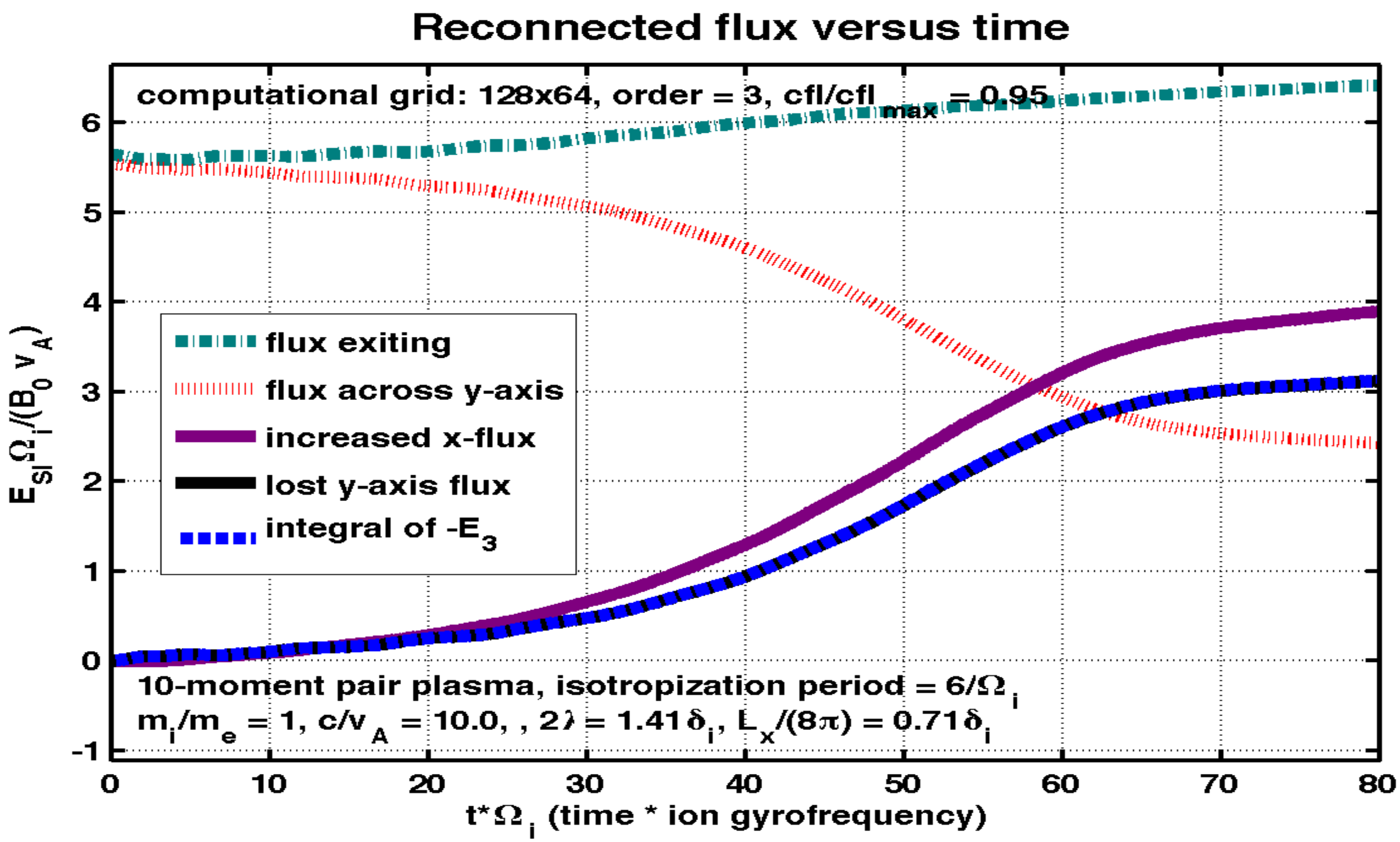
spatial unit = ion inertial length =δ_i, sheet width = 1.4142*δ_i


B at t = 76/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 78/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi


B at t = 80/Ω i (128x64 grid, isotropization period = 6/Ω i)
4
2
0
-2
-4
-8
-6
-4
-2
0
2
4
6
8
spatial unit = ion inertial length =δi, sheet width = 1.4142*δi

Magnetic reconnection: GEM problem



GEM magnetic reconnection challenge problem

Original GEM problem study in 2001:[1]

- Initiates reconnection by pinching adjacent oppositely directed field lines.
- Studied using particle and fluid models.
- Identified the Hall effect as critical for fast reconnection.
- Prompted study of reconnection in pair plasma (for which the Hall term vanishes).

BIRN ET AL.: GEM RECONNECTION CHALLENGE

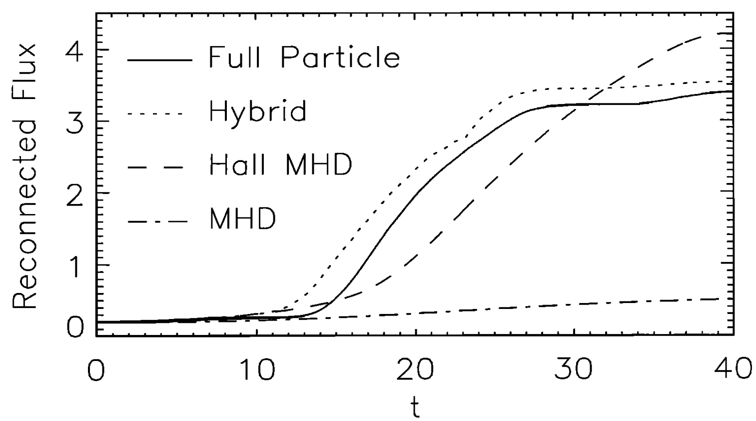


Figure 1. The reconnected magnetic flux versus time from a variety of simulation models: full particle, hybrid, Hall MHD, and MHD (for resistivity $\eta = 0.005$).

[1] Birn et al. Geospace environmental modeling (GEM) magnetic reconnection challenge. Journal of Geophysical Research Space Physics, 106:37153719, 2001.

Pair plasma

In 2005 Bessho and Bhattacharjee simulated fast reconnection in a pair plasma version of the GEM problem using a kinetic model.[2]

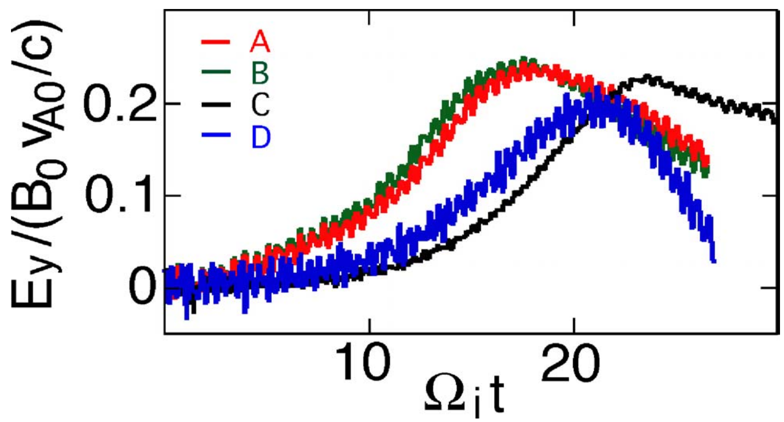


FIG. 2. (Color online) Reconnection electric fields as a function of time.

(Red curve is rate of reconnection for GEM-like pair plasma problem.)

Fluid models are computationally cheaper than kinetic models.

Can we find a simple fluid model of (pair) plasma that gives fast reconnection?

[2] N. Bessho and A. Bhattacharjee. Collisionless reconnection in an electron-positron plasma. Phys. Rev. Letters, 95:245001, December 2005.

Plasma Theory: GEM problem

There are only three sources that can provide for magnetic reconnection in any plasma model.

At the X-point, "Ohm's law" says that the rate of reconnection is the sum of a *resistive term*, a *nongyrotropic pressure term*, and an *inertial term*:

$$\text{rate of reconnection} = \mathbf{E}_3(0) = \left[\frac{-\mathbf{R}_i}{en_i} + \frac{\nabla \cdot \mathbb{P}_i}{en_i} + \frac{m_i}{e}\partial_t \mathbf{u}_i\right]_3\Big|_{\text{origin}}.$$

Consequences:

① For *steady-state* reconnection without resistivity the *pressure* term must provide for the reconnection.

② For a *gyrotropic* plasma without resistivity the *inertial* term must provide for the reconnection; i.e. each species velocity at the origin should track exactly with reconnected flux.

Fluid models of pair plasma

Magnetized pair plasma

Chacon et al.[3] obtained an analytical fluid model for fast reconnection in magnetized pair plasma (no null point). Viscosity provides the needed pressure anisotropy.

Unmagnetized pair plasma

Anomalous resistivity provides a way to cook up a desired rate of reconnection in pair plasma. One defines an anomalously high value of resistivity near the X-point.

- One-fluid models (i.e. MHD) make no assumption about mass ratio and can give fast reconnection when equipped with an anomalous resisivity
- Zenitani et al. have simulated fast reconnection in two-fluid five-moment models of collisionless pair plasma using anomalous resistivity. [4]
- We avoid anomalous resistivity and seek an uncontrived fluid model based on simple physical assumptions rather than problem-specific simulation results.

[3] L. Chacón, Andrei N. Simakov, V.S. Lukin, and A. Zocco. Fast reconnection in nonrelativistic 2D electron-positron plasmas. Phys. Rev. Letters, 101:025003, July 2008.

[4] S. Zenitani, M. Hesse, and A. Klimas. Two-fluid magnetohydrodynamic simulations of relativistic magnetic reconnection. *The Astrophysical Journal*, 696:13851401, May 2009.

Fluid models: five-moment and ten-moment models

Hakim et al. studied the GEM reconnection challenge problem using a two-fluid model with five moments for the electron fluid and five[5] or ten[6] moments for the ion fluid.

The five moment model and the ten-moment model are two hyperbolic models which fail on the GEM problem in different ways for different reasons:

① The five-moment model cannot reconnect without (numerical, anomalous) resistivity.

② The ten-moment model fails to (reliably) reconnect due to undamped oscillatory exchange between the inertial and pressure terms of Ohm's law.

Key observation: the five-moment model is the ten-moment model instantaneously relaxed to isotropy.

So we can get an intermediate model by slowing down the rate of isotropization.

This intermediate model appears to have neither of the problems of the two extreme models for a large range of isotropization rates.

---

[5] A. Hakim, J. Loverich, and U. Shumlak. A high-resolution wave propagation scheme for ideal two-fluid plasma equations. J. Comp. Phys., 219:418442, 2006.

[6] A.H. Hakim. Extended MHD modelling with the ten-moment equations. J. Fusion Energy, 27(12):3643, June 2007.

Our fluid model: relaxation toward isotropy

We implemented a two-fluid anisotropic model with relaxation toward isotropy.

We applied this model to pair plasma (a hot topic, a simple and illuminating singular case, and computationally cheap).

For a broad intermediate range of isotropization rates we obtain fast reconnection with dominant contribution from the pressure term, in agreement with particle simulations and the theory of steady-state non-resistive reconnection. For extreme rates of isotropization the model behavior is unreliable.

Modeling equations: ten-moment two-fluid-Maxwell model

Generic physical equations for the ten-moment two-fluid model are: (1) conservation of mass and momentum and pressure tensor evolution for each species:

$$\partial_t \rho_s + \nabla \cdot (\rho_s \mathbf{u}_s) = 0,$$

$$\partial_t (\rho_s \mathbf{u}_s) + \nabla \cdot (\rho_s \mathbf{u}_s \otimes \mathbf{u}_s + \mathbb{P}_s) = \frac{q_s}{m_s} \rho_s (\mathbf{E} + \mathbf{u}_s \times \mathbf{B}) + \mathbf{R}_s,$$

$$\partial_t \mathbb{P}_s + \nabla \cdot (\mathbf{u}_s \mathbb{P}_s) + 2\,\mathrm{Sym}\,(\mathbb{P}_s \cdot \nabla \mathbf{u}_s) + \nabla \cdot \mathbb{Q}_s = 2\,\mathrm{Sym}\,(\frac{q_s}{m_s} \mathbb{P}_s \times \mathbf{B}) + \mathbb{R}_s,$$

and (2) Maxwell's equations for evolution of electromagnetic field:

$$\partial_t \mathbf{B} + \nabla \times \mathbf{E} = 0, \qquad \nabla \cdot \mathbf{B} = 0,$$

$$\partial_t \mathbf{E} - c^2 \nabla \times \mathbf{B} = -\mathbf{J}/\epsilon, \qquad \nabla \cdot \mathbf{E} = \sigma/\epsilon.$$

We assume that $\mathbf{R}_s = 0$, and to provide for isotropization we let $\mathbb{R}_s = \frac{1}{\tau_s}\left(\frac{1}{3}(\mathrm{tr}\,\mathbb{P}_s)\mathbb{I} - \mathbb{P}_s\right)$;
In our present work we assume that $\mathbb{Q}_s = 0$. We implemented a second-order and third-order discontinuous Galerkin two-fluid solver.

Results

We varied the rate of isotropization from zero to instantaneous and simulated the GEM magnetic reconnection challenge problem for a pair plasma.

We plotted the contribution of proxy Ohm's law terms to the reconnected flux.

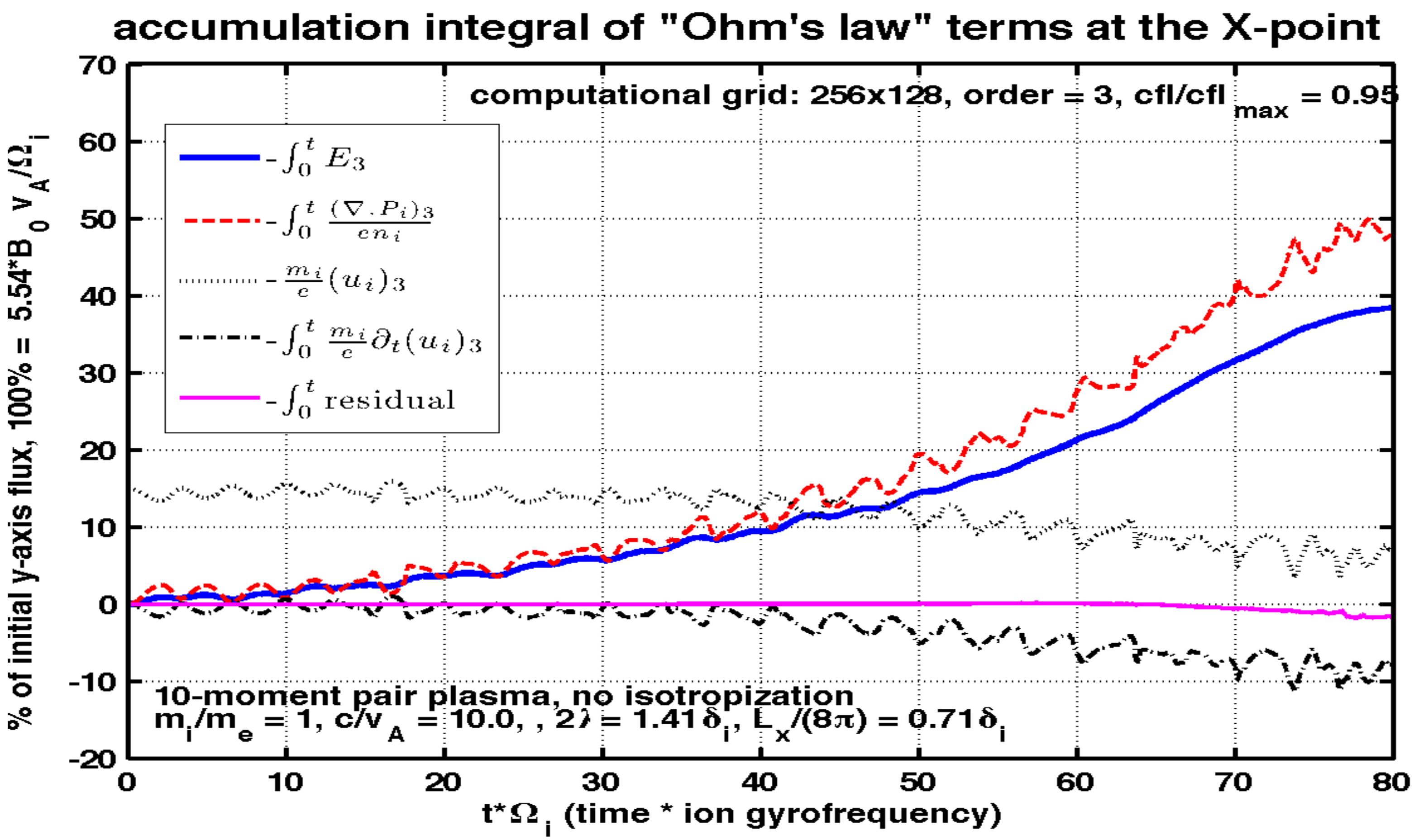
We find that:

① **for intermediate isotropization rates:** reconnection is fast and that the pressure term makes the dominant contribution to reconnected flux, in agreement with PIC simulations and steady-state reconnection theory,

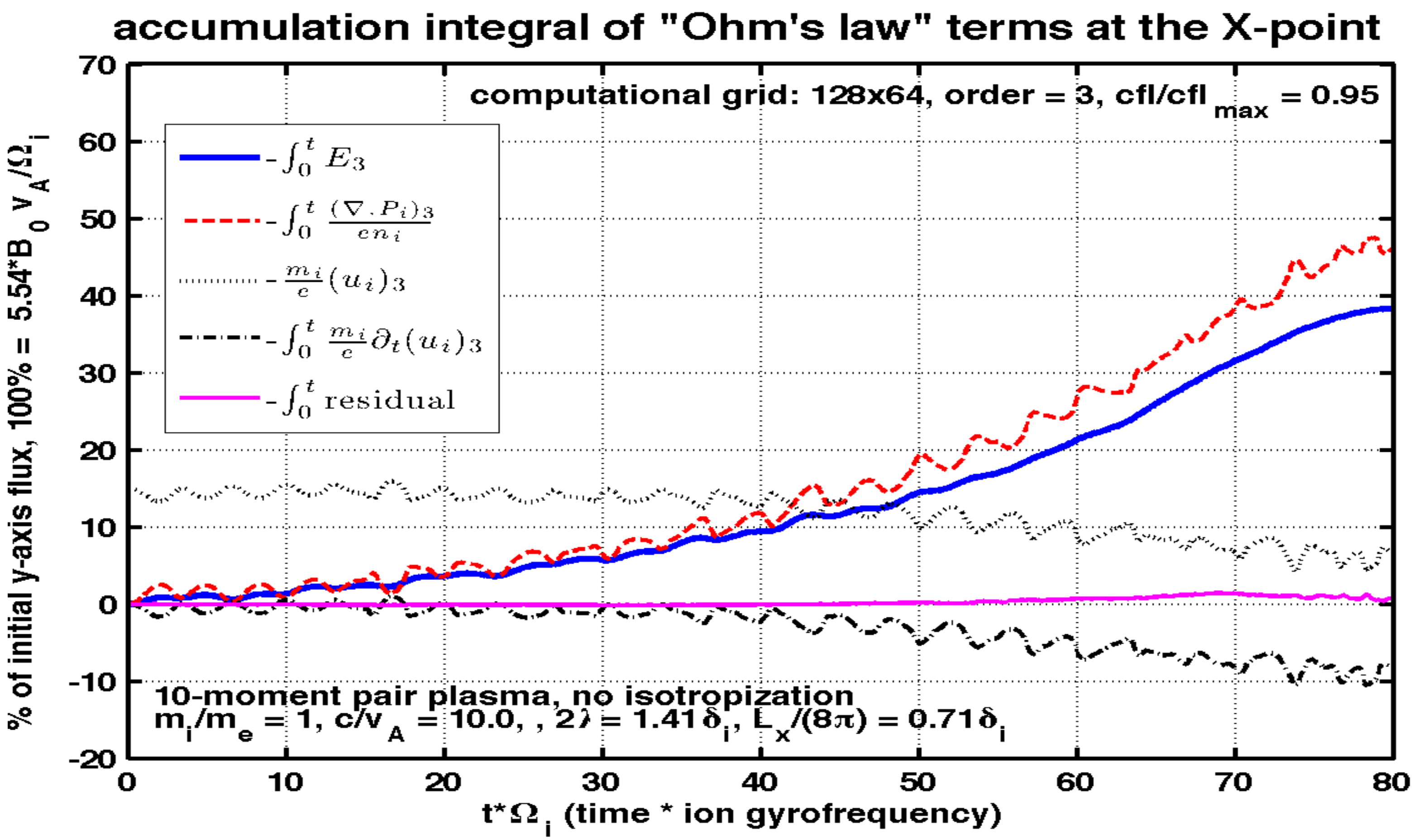
② *for very fast isotropization rates:* fast reconnection begins and current tracks with reconnected flux, but numerical instability kicks in and numerical resistivity takes over, and

③ *for very slow isotropization rates:* our simulations are not refined enough to determine reconnection rates. Undamped oscillatory exchange between the pressure and inertial term seems to occasion instability.

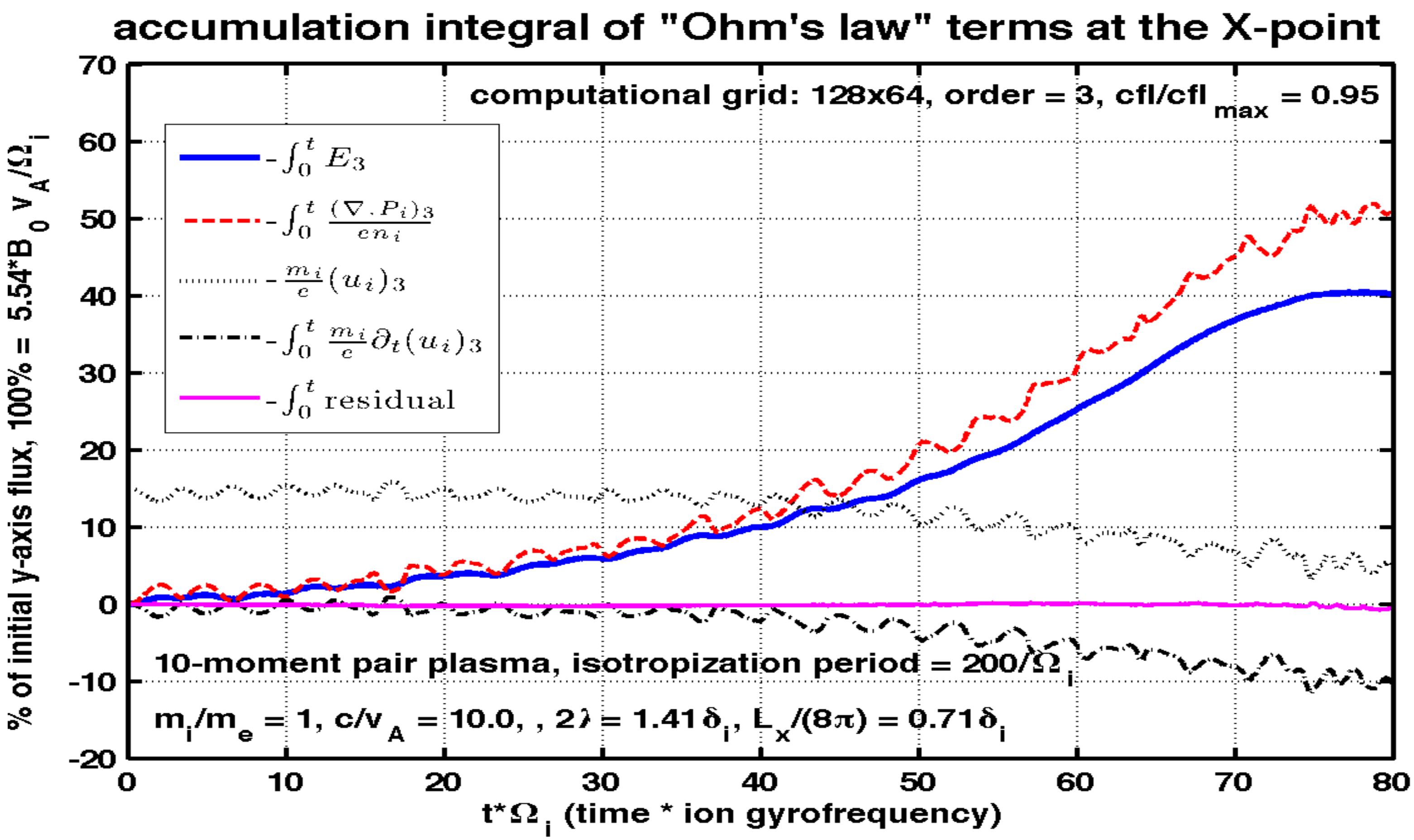
Results: reconnection for no isotropization, fine mesh



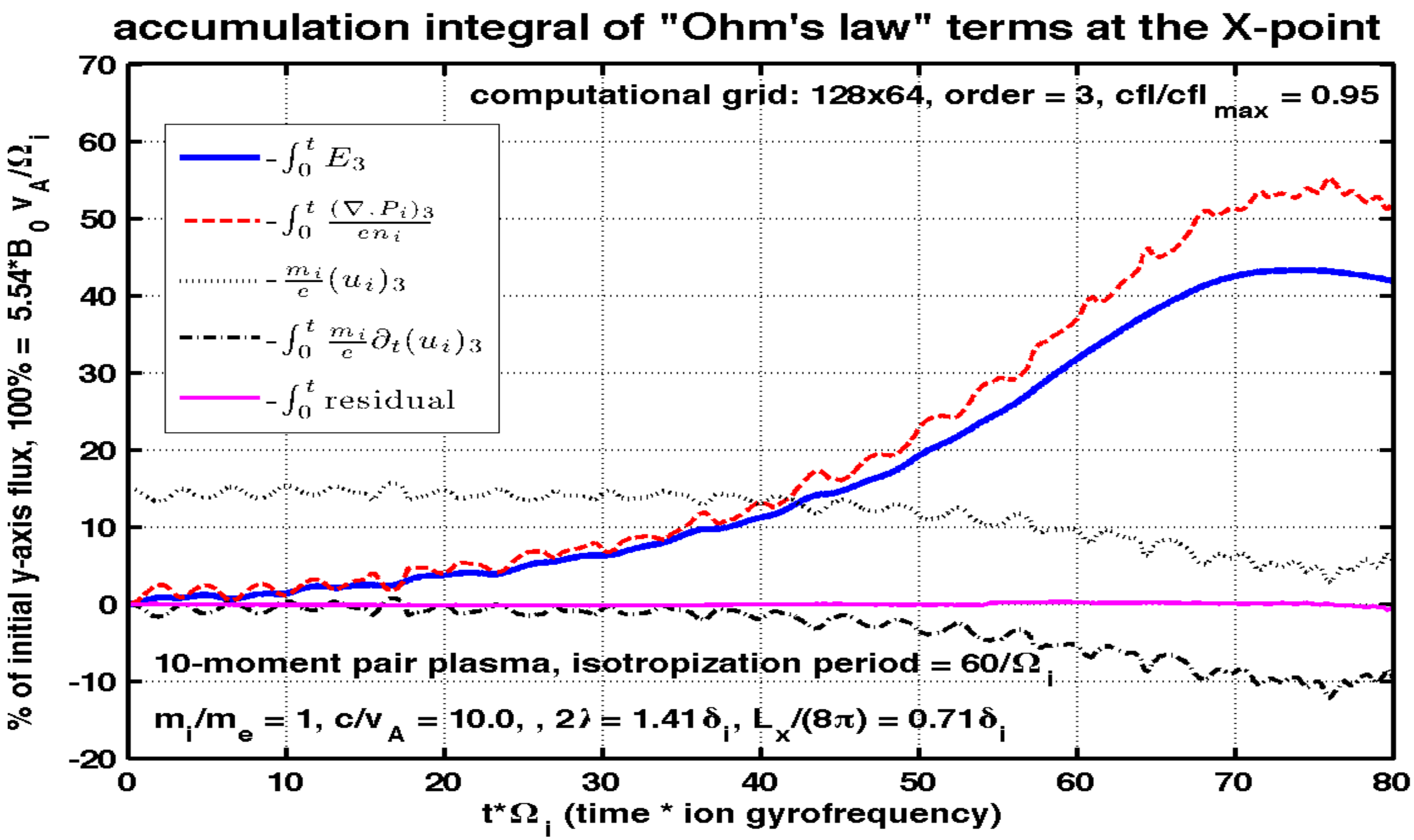
Results: reconnection for no isotropization



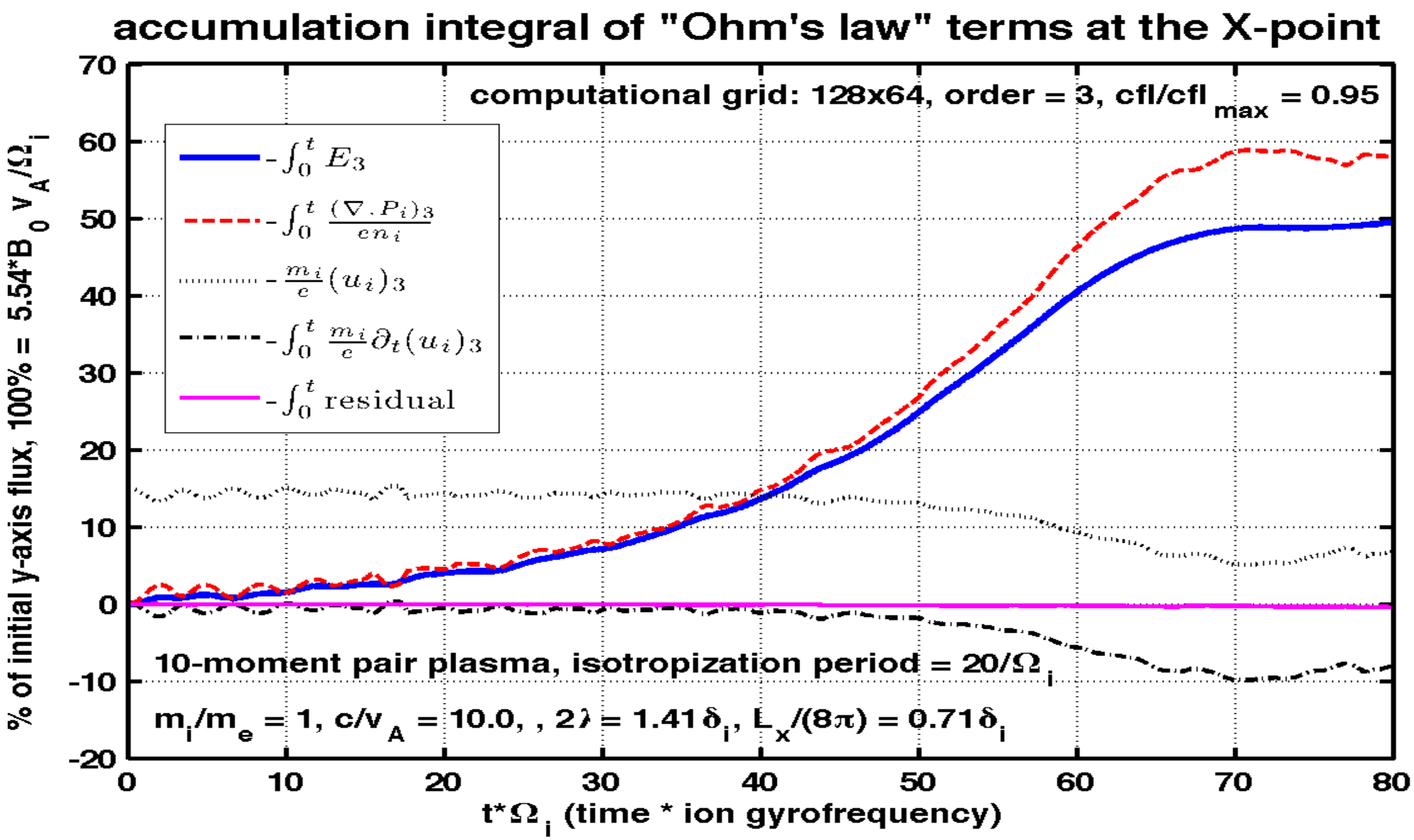
Results: reconnection for extremely slow isotropization



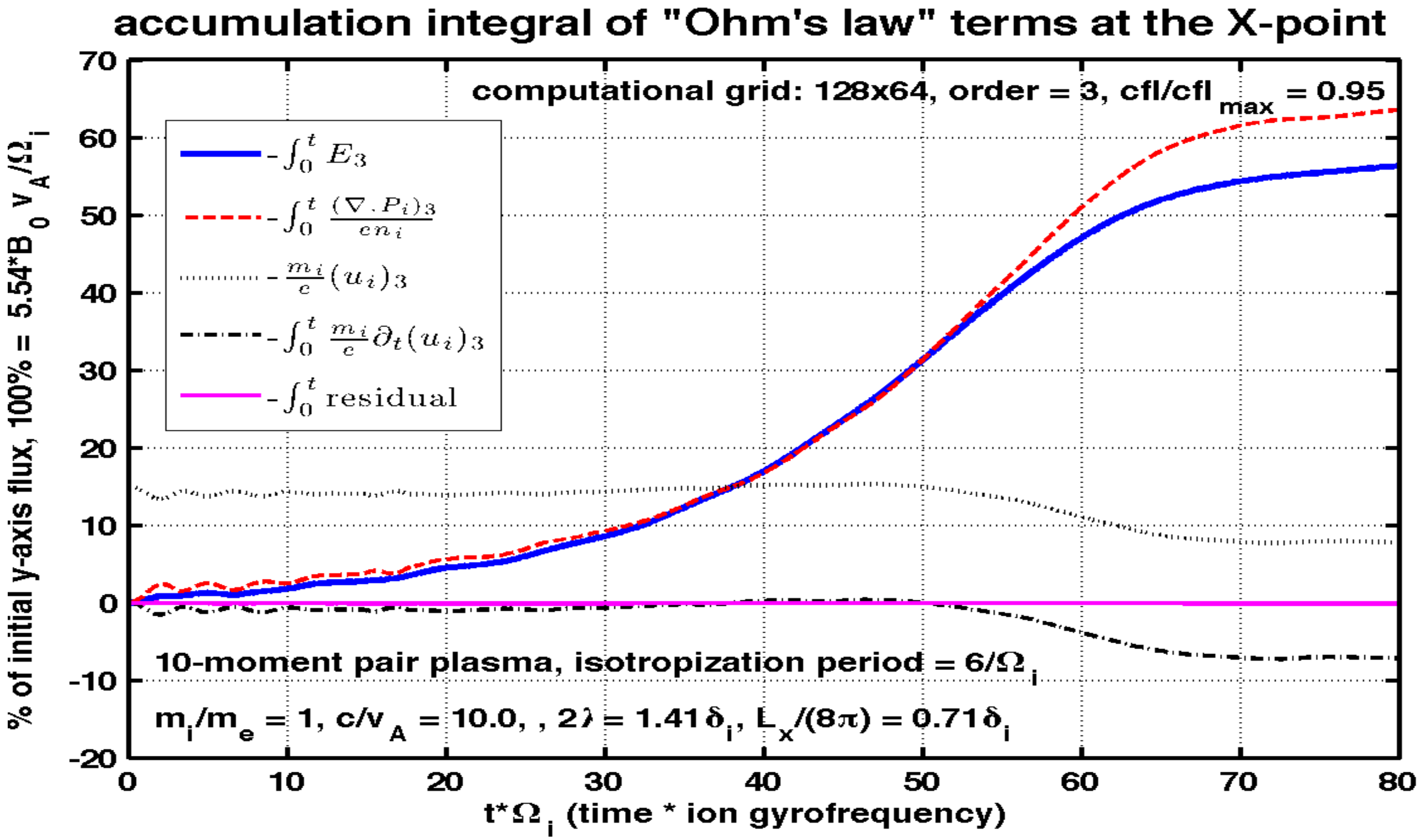
Results: reconnection for very slow isotropization



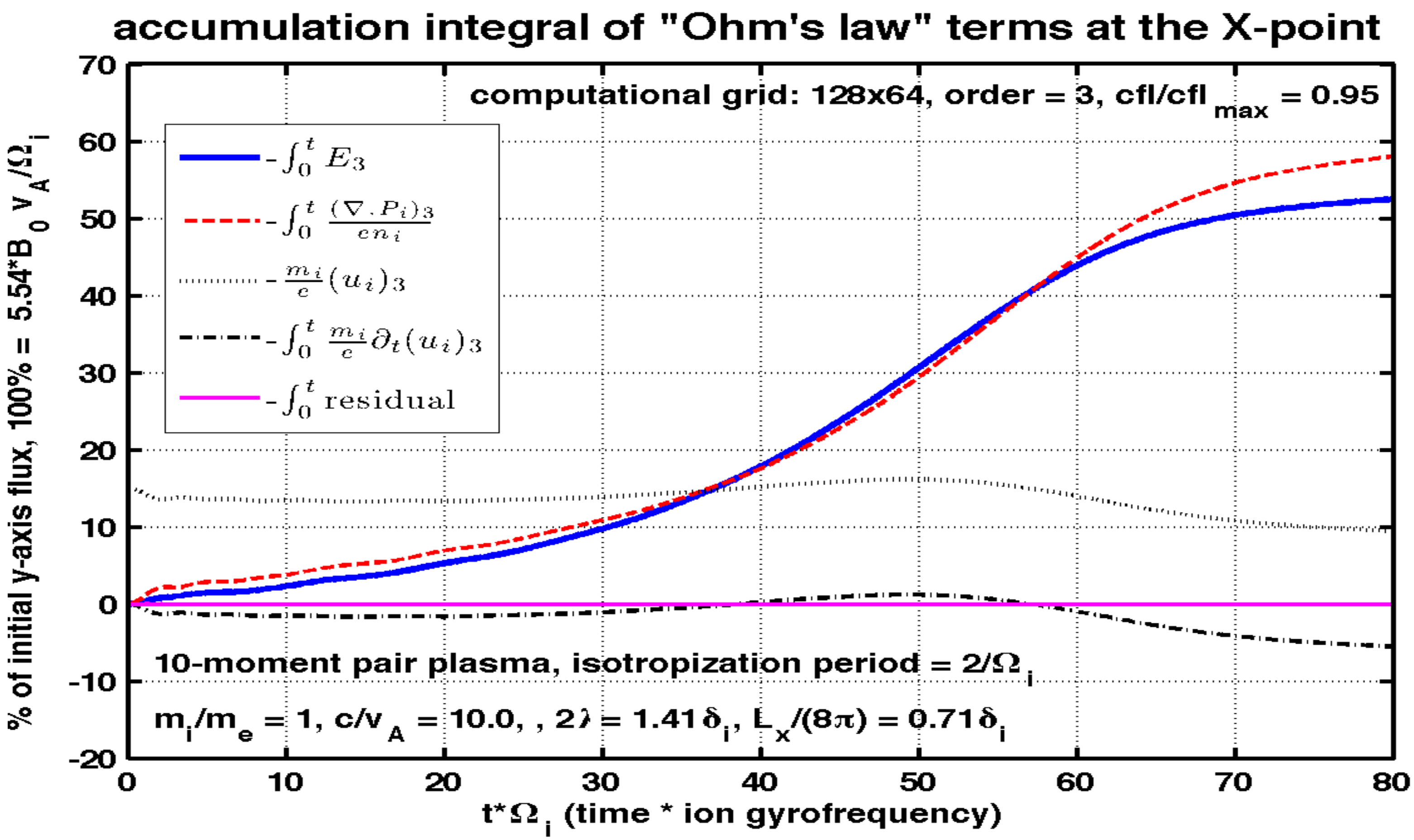
Results: reconnection for slow isotropization



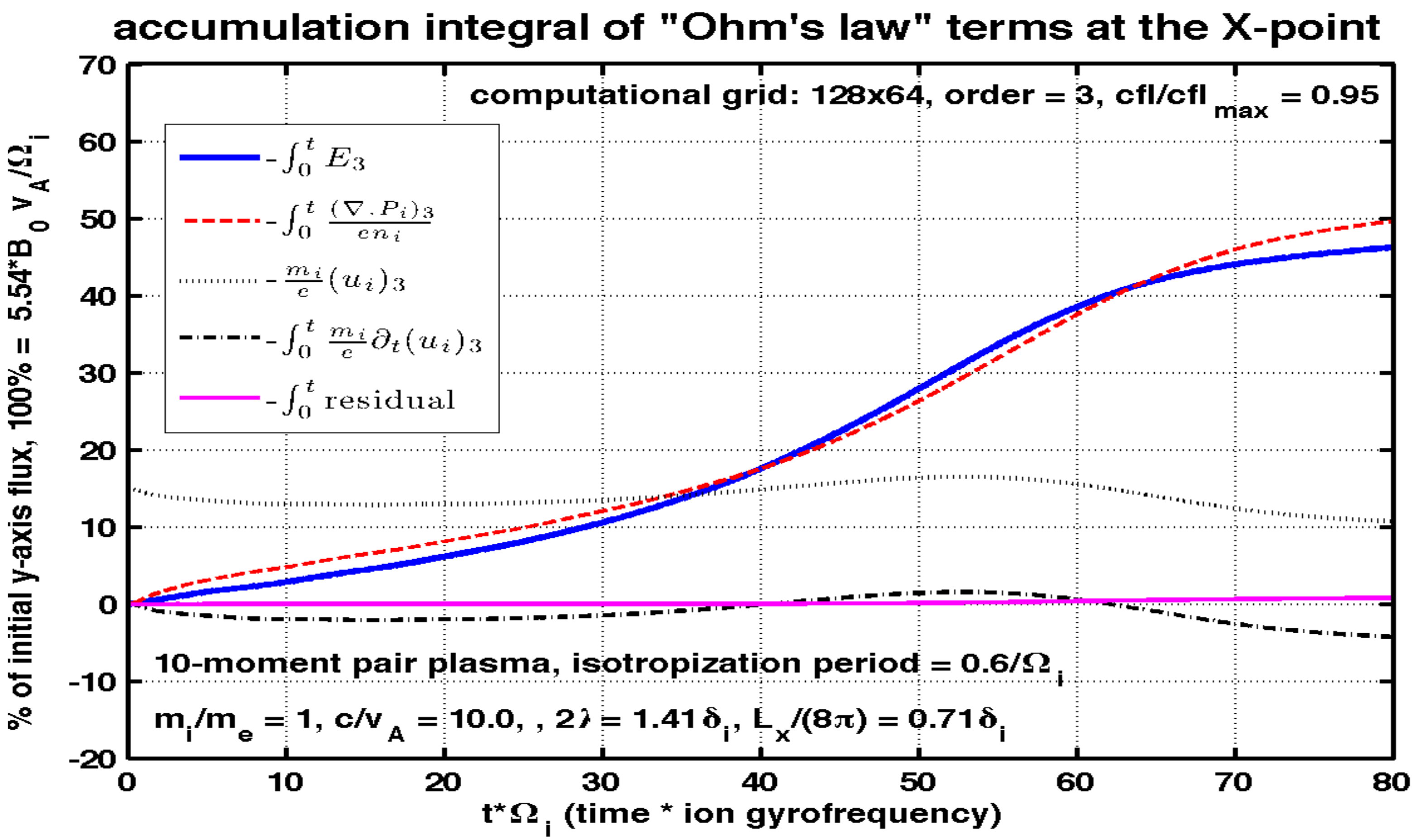
Results: reconnection for slow intermediate isotropization



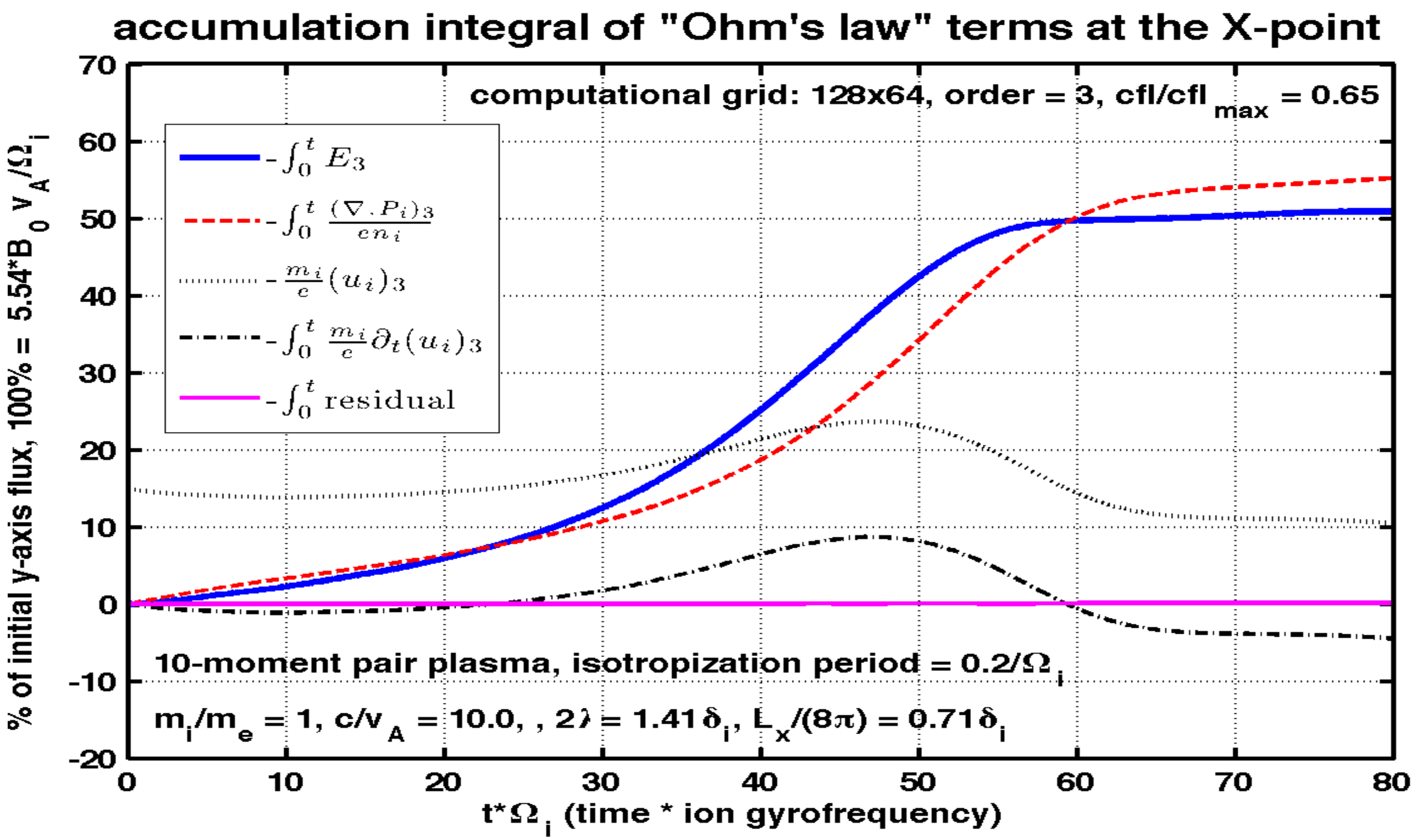
Results: reconnection for intermediate isotropization



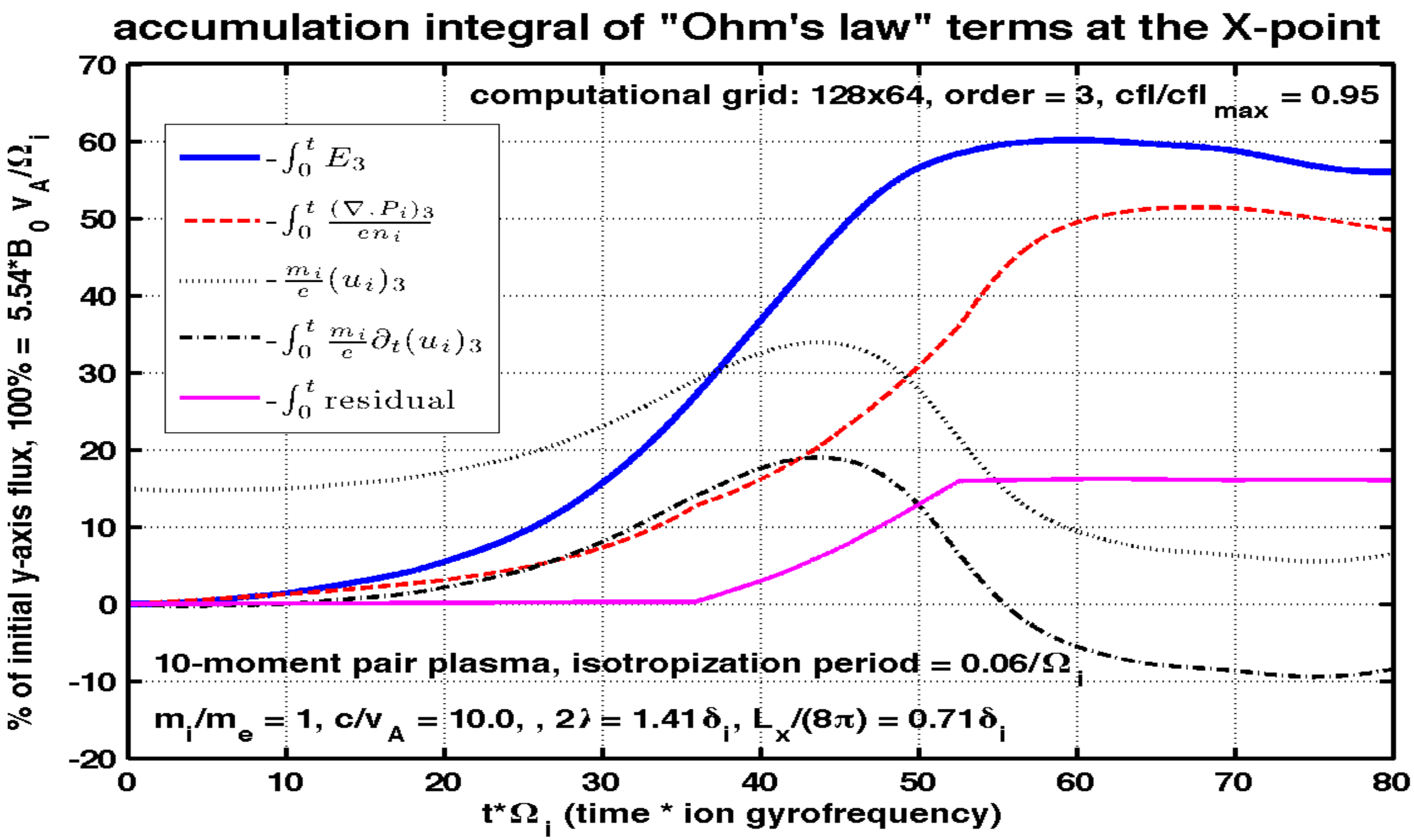
Results: reconnection for fast intermediate isotropization



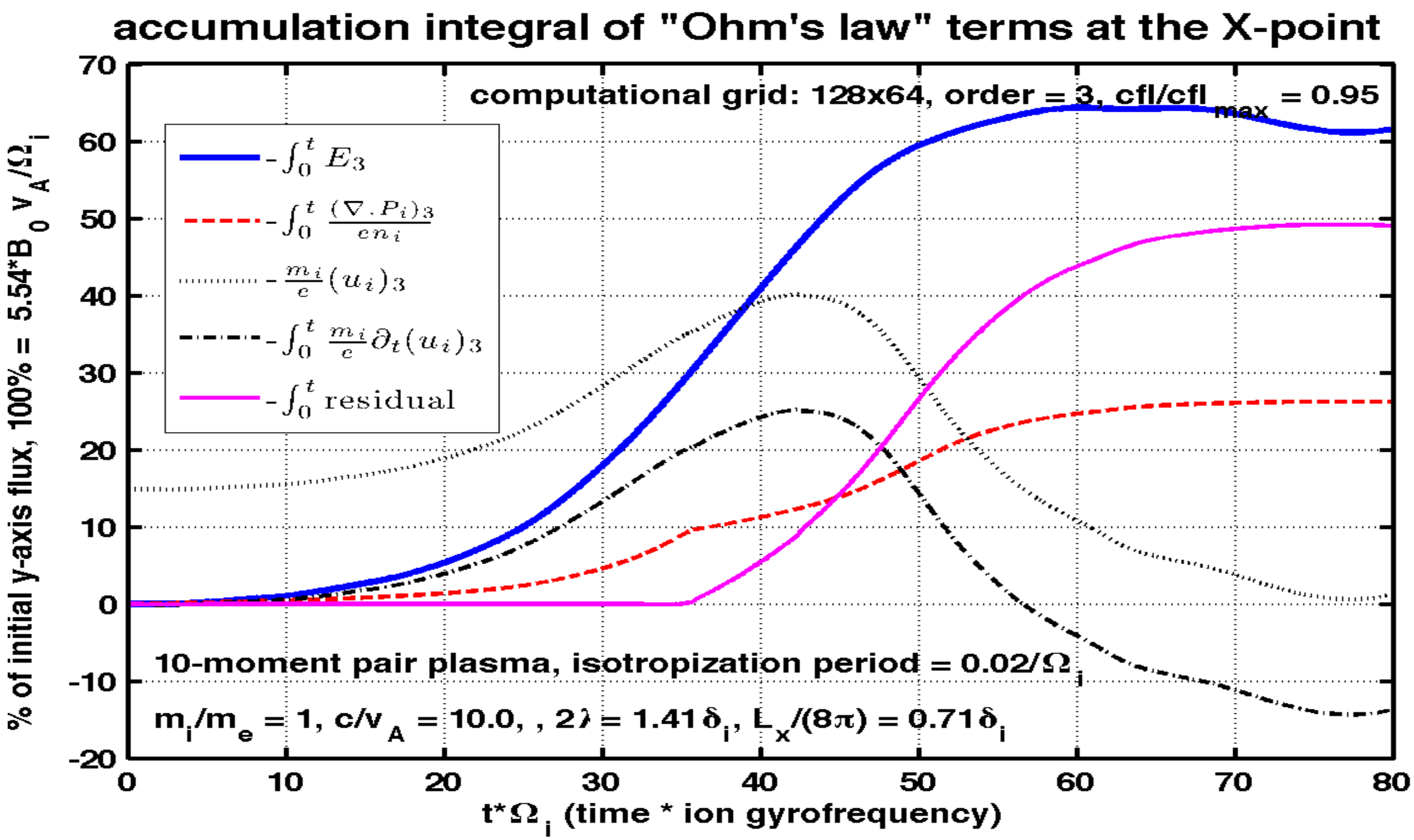
Results: reconnection for fast isotropization



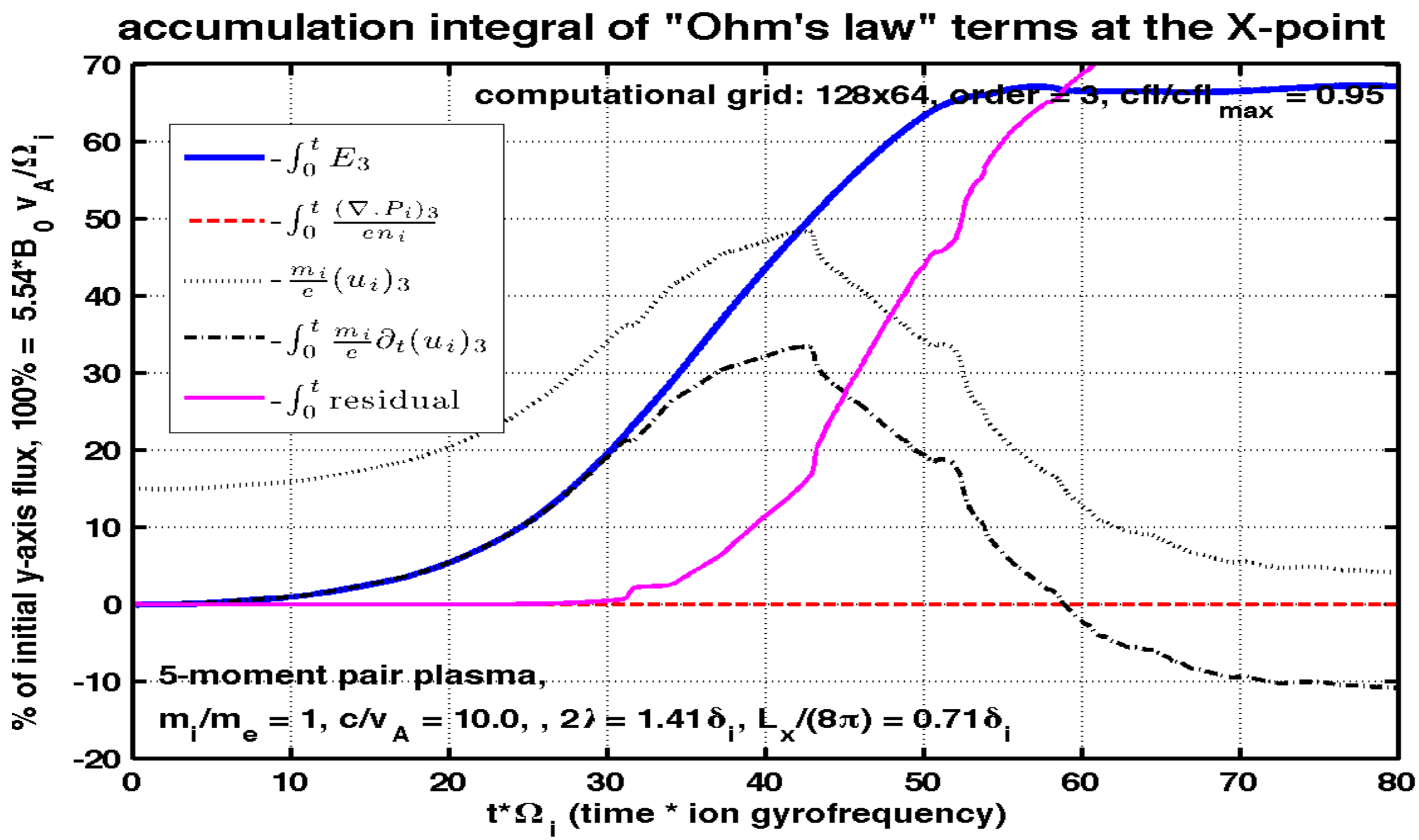
Results: reconnection for very fast isotropization



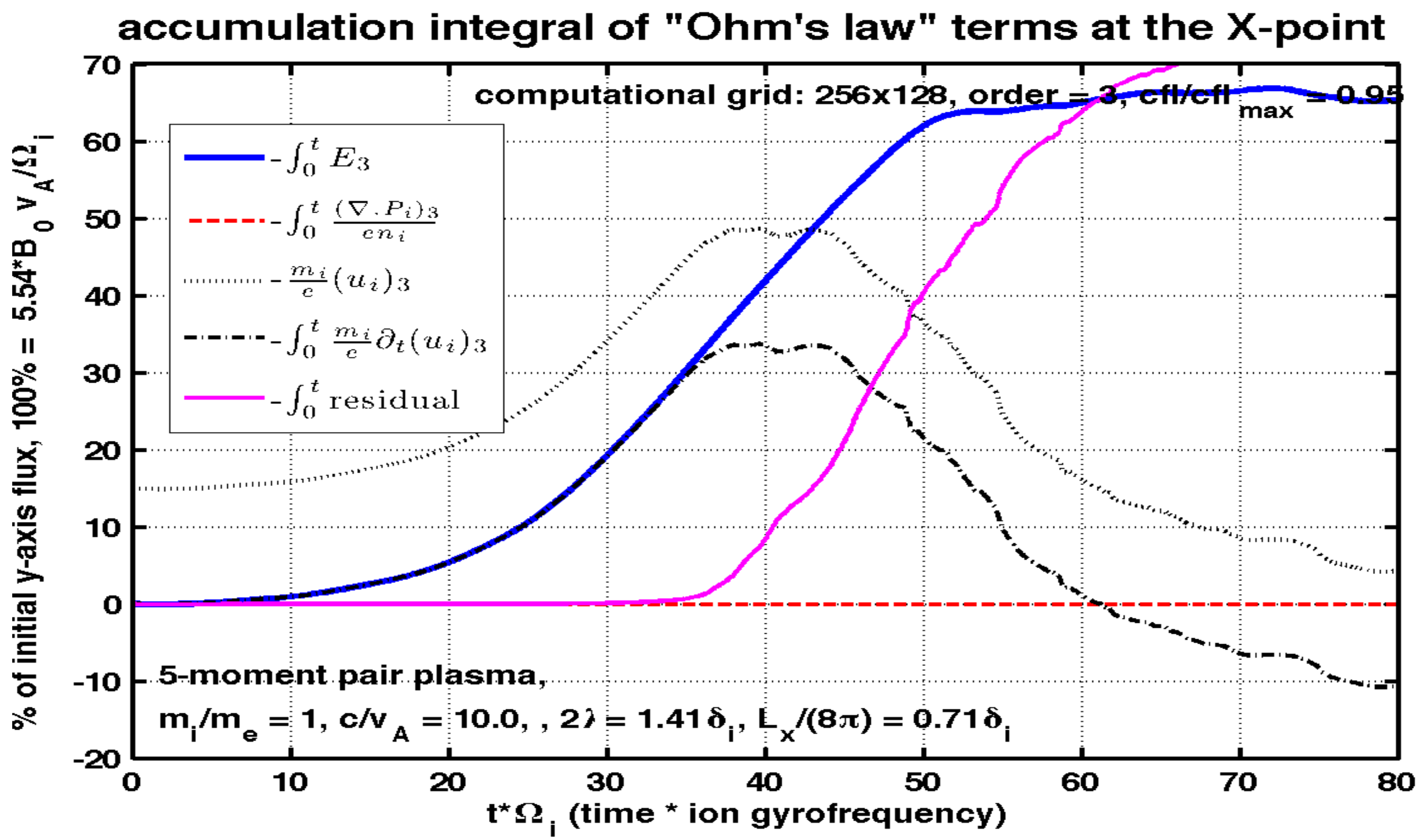
Results: reconnection for extremely fast isotropization



Results: reconnection for instantaneous isotropization



Results: reconnection for instantaneous isotropization, fine mesh.



GEM settings: mismatch of reconnection rate

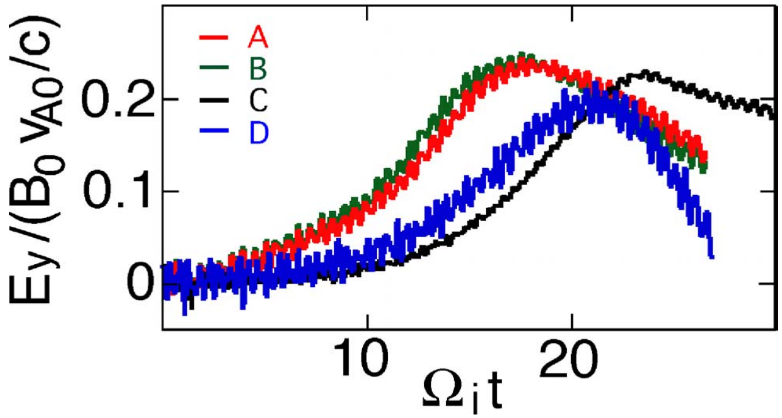
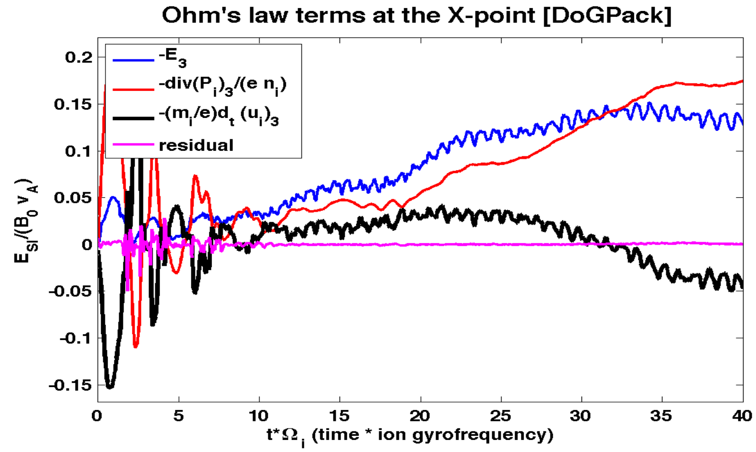
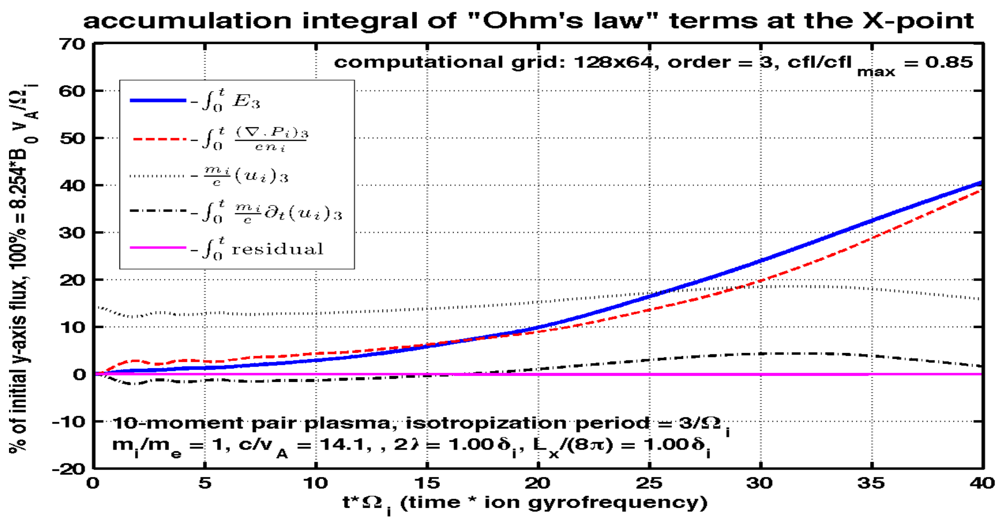


FIG. 2. (Color online) Reconnection electric fields as a function of time.





Future Work

Plans for futher investigation:

① Particle studies indicate that nonzero heat flux is necessary to get pressure nongyrotropy [7]. So we hope to get fast reconnection without isotropization by using C. David Levermore's closure for the heat flux tensor [8],

$$\mathbb{Q}_s = \frac{9}{5}(\nu_0 - \nu_1)\mathbb{I} \vee \mathrm{tr}\left(\nabla \vee \Theta_s^{-1}\right) + 3\nu_1\Big(\nabla \vee \Theta_s^{-1}\Big).$$

② We hope to extend our work to relativistic pair plasma (the most common pair plasma regime of interest).

③ We hope to generalize to nonzero guide field and non-pair plasmas.

[7] M. Hesse, M. Kuznetsova, and J. Birn. The role of electron heat flux in guide-field magnetic reconnection. Physics of Plasmas, 11(12):53875397, 2004.

[8] C. David Levermore. Kinetic theory, Gaussian moment closures, and fluid approximations. Presented at IPAM KT2009 Culminating Retreat, Lake Arrowhead, California, June 2009.

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- My advisor, James Rossmanith
- Nick Murphy, Ellen Zweibel, Ping Zhu