Fluid exercises

The goal of these exercises is to derive the system of equations listed at the top of the Waves in MHD handout that I distributed at the end of class last time. This should prepare you to derive the three simplest MHD waves that I mentioned (Alfvén, sound, and fast magnetosonic waves). The system of equations at the top of that handout is:

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0,$$
$$\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) + \nabla p = \mu_0^{-1} (\nabla \times \mathbf{B}) \times \mathbf{B},$$
$$\partial_t p + \mathbf{u} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{u} = 0,$$
$$\partial_t \mathbf{B} + \nabla \times (\mathbf{B} \times \mathbf{u}) = 0,$$

where I have used that $\mu_0 \mathbf{J} = \nabla \times \mathbf{B}$.

I am in the United States from now until the end of the semester, so if you have questions you should email me at `Alec.Johnson@wis.kuleuven.be` *or talk to one of the other instructors.*

Problem 1

Derive the following equations.

- mass conservation (again)
- momentum balance (again)
- energy balance

You may neglect the deviatoric pressure, the heat flux, and the collisional drag (**r** in the notes).

Problem 2

Following the slides from the lecture, derive pressure balance. This proceeds in three steps:

1. Dot momentum balance with the fluid velocity $\mathbf{u}$ to get balance of the kinetic energy $\mathcal{E}_{\text{kinetic}} := \frac{1}{2}\rho \mathbf{u} \cdot \mathbf{u}$.
2. Subtract kinetic energy balance from energy balance to get balance of the thermal energy $\mathcal{E}_{\text{thermal}} := \mathcal{E} - \mathcal{E}_{\text{kinetic}}$.
3. rewrite thermal energy balance as balance of the pressure $p := \frac{2}{3}\mathcal{E}_{\text{thermal}}$.

Problem 3

Make MHD simplifications:

- Assume that $\partial_t \mathbf{E} = 0$. What closure does this give you for $\mathbf{J}$?
- Assume Ohm's law: $\mathbf{E} = \mathbf{B} \times \mathbf{u}$. (This says that the electric field is zero in the reference frame of the fluid.)
- Write out the full MHD system.