Exercises for Section 1.1

1. Let $R = \{a, b\}$, $S = \{a, c, f\}$, and $T = \{a, b, c, d, e\}$. Decide whether each of the following assertions is correct.
   (a) $S \subset T$ (b) $R \subset S$ (c) $b \in R \cap T$

2. With R, S, and T defined as in Exercise 1, decide whether each of the following assertions is correct.
   (a) $R \subset T$ (b) $(R \cup S) \subset T$ (c) $c \in S \cap T$

3. With R, S, and T defined as in Exercise 1, find $(R \cup S) \cap T$.

4. Let $U = \{1, 2, t, u, v, x, y, z\}$, $E = \{2, t, y\}$, $F = \{1, 2, u, y, z\}$, and $G = \{1, 2, u, y\}$. Find $(E \cup F) \cap G'$.

5. Let $A = \{p, q, r\}$. Find all nonempty subsets B and C of A such that $B \cap C = \varnothing$ and $B \cup C = A$.

6. Let $U = \{1, 2, 3, 4, 5\}$ be a universal set with subsets $A = \{2, 4\}$ and $B = \{1, 5\}$. List the elements in each of the following sets.
   (a) A' (b) $A' \cap B'$ (c) $A \cup B$ (d) $(A \cup B)'$

7. The sets M, A, B, and C are defined as follows:

$$M = \{\text{Minnesota, Michigan, Montana, Massachusetts}\}$$
$$A = \{\text{Alabama, Arkansas, Michigan}\}$$
$$B = \{\text{Montana, Michigan}\}$$
$$C = \{\text{Alabama, Arkansas}\}$$

   Decide which of the following subset relationships are correct.
   (a) $B \subset M$ (b) $B \subset C$ (c) $C \subset A$
   (d) $C \subset B$ (e) $C \subset M$ (f) $A \subset (B \cup C)$

8. The sets R, S, and T are subsets of a universal set U. Which of the following always holds?
   (a) $R \cap S \subset R$ (b) $T \subset T \cap \varnothing$
   (c) $R \cup (S \cap T) \subset R \cap (S \cup T)$ (d) $R' \cup S' = (R \cup S)'$

9. Let $U = \{u, v, w, x, y, z, 1, 2, 3\}$, $E = \{2, y, w, z\}$, $F = \{2, 3, u, y, z\}$, and $G = \{1, 2, 3, w, y\}$. List the elements in each of the following sets.
   (a) E' (b) $F \cup G'$ (c) $(E \cup F) \cap G'$

10. Let $U = \{x, y, z, 1, 2, 3\}$, $A = \{y, z, 2\}$, $B = \{y, 1, 2\}$, and $C = \{x, 3\}$. List the elements in each of the following sets.
   (a) $A \cup B$ (b) $B \cap C$ (c) A'
   (d) $(A \cup B) \cap (B \cup C)$ (e) $(B \cap A') \cap C'$

11. Let U, A, B, and C be defined by

$$U = \{a, b, c, 1, 2, 3\}$$
$$A = \{a, b, c\} \qquad B = \{a, 2, 3\} \qquad C = \{1, 2, 3\}$$

   List the elements in each of the following sets.
   (a) $A \cup B$ (b) $B \cap C$ (c) $(A \cup B) \cap (B \cup C)$
   (d) A' (e) $A \cap B'$ (f) $A \cup C'$

12. Let $X = \{b, p, 4, 7\}$ and $Y = \{a, p, 4\}$ be subsets of a universal set $U = \{a, b, m, p, 1, 4, 7\}$. Which of the following is NOT a true statement?
   (a) $b \in X \cup Y$ (b) $\{p, 4\} = X \cap Y$ (c) $7 \in X \cap Y'$
   (d) $1 \in X' \cap Y$ (e) $1 \in X' \cup Y$

Business 13. Let sets A, B, C, and D be defined by

$$A = \{x : x \text{ owns a GM car}\}$$
$$B = \{x : x \text{ works for GM}\}$$
$$C = \{x : x \text{ is the president of GM}\}$$
$$D = \{x : x \text{ owns stock in GM}\}$$

Describe in words each of the following sets.
(a) $A \cap B$ (b) $B \cap A'$ (c) $(A \cup B) \cap D$ (d) $C \cap A$

14. Let X and Y be sets with $a \in X$ and $b \in Y$. Is it *always* true (yes or no) that $\{a, b\} \subset X \cup Y$? That $\{a, b\} \subset X \cap Y$?

15. Let A and B be subsets of a universal set U. It is *always* true that
(a) $B \cap A' \subset A$ (b) $A \cap B \subset A \cup B$
(c) $A' \cap B' \subset (A \cap B)'$ (d) $A' \cup B' \subset (A \cup B)'$

16. Let A, B, C, and D be subsets of U with $A \subset B$ and $C \subset D$. Is it *always* true that
(a) $A \cap C \subset B \cap D$ (b) $A' \cap C' \subset B' \cap C'$

17. Let $U = \{2, 4, 8, 16, 32, 64\}$. Which of the following pairs of subsets A, B, of the universal set U, satisfy the condition: $A \cap B' = \{2, 16\}$
(a) $A = \{2, 8, 16\}$, $B = \{4, 8, 64\}$
(b) $A = \{2, 16, 32\}$, $B = \{4, 8, 64\}$
(c) $A = \{2, 16\}$, $B = \{4, 64\}$

18. Let $U = \{w, x, y, z\}$. Find examples of subsets A and B of U which satisfy the stated condition.
(a) $A \cup B = A$ (b) $A \cap B = A$
(c) $A \cap B' = A$ (d) $A \cap B' = B \cap A'$

19. Let $U = \{1, 2, 3, 4, x, y\}$ be a universal set with subsets $X = \{1, 2, 3, x, y\}$, $Y = \{2, 4, y\}$, and $Z = \{2, x\}$. Use intersections, unions, and complements to express each of the following sets in terms of X, Y, and Z.

$$A = \{2, y\} \qquad B = \{1, 3, y\} \qquad C = \{2, 4, x, y\}$$

20. With X, Y, Z, and U as in Exercise 19, use intersections, unions, and complements to express the set $\{x\}$ in terms of X, Y, and Z.

21. List all subsets of the following sets.
(a) $\{x\}$ (b) $\{x, y\}$ (c) $\{x, y, z\}$

22. Counting the empty set and the set itself, how many subsets does each of the following sets contain?
(a) $\{x\}$ (b) $\{x, y\}$ (c) $\{x, y, z\}$ (d) $\{w, x, y, z\}$
Is there a pattern? If so, what is the pattern? How many subsets does a set with seven elements contain?

23. Let $U = \{a, b, c, 2, 4, 6\}$ be a universal set with subsets X, Y, and Z. Suppose that $X \cup Y = \{b, c, 2, 4, 6\}$, $X \cap Y = \{b, 2, 4\}$, $Y' \cap Z' = \{a, c\}$, and $Z' = \{a, c, 2\}$. Find sets X, Y, and Z which satisfy these conditions.

24. If $A = \{r, s, t\}$ and $B = \{s, t, u\}$: list the elements in $B \times A$.

25. Let $A = \{a, b, c\}$ and $B = \{a, b, d\}$.
(a) List the elements in $A \times B$.
(b) List the elements in $(A \times B) \cap (B \times A)$.

26. Let $U = \{1, 2, 3, 4, 5, 6\}$ be a universal set with subsets X, Y, and Z. Suppose that $X \cup Y = \{1, 2, 4\}$, $Y \cap Z = \{4\}$, $(Y \cup Z)' = \{1, 3, 5\}$, $X \cap Y = \{4\}$, and $Z' = \{1, 2, 3, 5\}$. Find subsets X, Y, and Z.

27. Suppose $A \times B = \{(a, 1), (b, 1), (a, 2), (b, 2), (a, 3), (b, 3)\}$. Find A and B.

28. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 4\}$. Decide which of the following are correct.
    (a) $(A \times B) \cap (B \times A) = (A \times A) \cap (B \times B)$
    (b) $(A \times B) = \{(2, 1), (1, 4), (1, 2), (3, 4), (3, 2), (1, 1), (2, 2), (2, 4), (3, 1)\}$
    (c) $(A \times A) \subset ((A \times B) \cap (B \times A))$
    (d) $(A \times A) \subset ((A \times B) \cup (B \times A))$

29. Let $U = \{-2, -1, 0, 1, 2\}$ and $S = \{-1, 0, 1\}$. Also, let $A = \{(x, y) : x \in S, y = x^2\}$ and $B = \{(x, y) : x \in U, y = x^2\}$. Is it true that
    (a) $A \subset B$ (b) $A \subset S \times S$ (c) $B \subset S \times S$ (d) $B \subset U \times U$

30. Let A, B, S, and U be the sets given in Exercise 29. Find $A \times A$ and $B \times B$. Show that $A \times A \subset B \times B$. Is it always true that if $A \subset B$, then $A \times A \subset B \times B$? Why or why not?

31. Suppose $A \times B = \{(1, a), (1, b), (1, c), (2, a), (2, b), (2, c)\}$. If $C = \{d, e\}$, then $C \times A = ?$

32. Let $A = \{1, 2, 3, 4, 6, 8\}$ and $B = \{5, 6, 7, 8, 9\}$. A set W is defined to be the elements (pairs) of $B \times A$ for which at least one of the numbers is even. How many elements are there in W?

33. Let $A = \{x, y, z, 1\}$ and $B = \{1, 2, 4\}$. If a set $U = (A \times B) \cup (B \times A)$, how many elements are there in U?