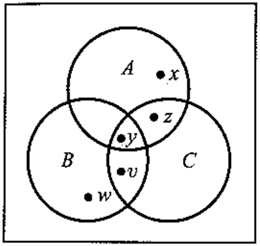
Exercises for Section 1.2

1. Let A, B, and C be subsets of a set U. Draw a Venn diagram to illustrate each of the following sets. In each case, shade the area corresponding to the designated set.
   (*a*) $A' \cap B$ (*b*) $A \cup B'$
   (*c*) $A \cup B \cup C$ (*d*) $(A \cup B) \cap C'$

FIGURE 1.5



2. In each case determine which of points v, w, x, y, and z in Figure 1.5 are contained in the specified set.
   (*a*) $A \cap C'$ (*b*) $A \cup C'$
   (*c*) $A \cup (B \cap C)$ (*d*) $(B \cap C)'$
3. In each case determine which of points v, w, x, y, and z in Figure 1.5 are contained in the specified set.
   (*a*) $A \cup C$ (*b*) $A \cap B$
   (*c*) $A \cup B$ (*d*) $B \cap C$
4. Using Figure 1.5, decide which of the following statements are true and which are false.
   (*a*) $z \in A \cup C'$ (*b*) $y \in B \cup (A \cap C')$
   (*c*) $y \in (B \cup C) \cap A'$ (*d*) $v \in (B \cup C) \cap (A \cup C)$
5. Using Figure 1.5, decide which of the following statements are true and which are false.
   (*a*) $\{x, y\} \subset A \cap B \cap C$ (*b*) $\{v, y, z\} \subset (A \cap C) \cup B$
   (*c*) $\{w, x, y\} \subset (A \cup B) \cap C$ (*d*) $\{y, z, v\} \subset (A \cup B) \cap C$
6. Describe the shaded areas in each Venn diagram of Figure 1.6, by using the set operations of union, intersection, and complement and the sets A, B, and C.
7. Repeat Exercise 6, using Figure 1.7.
8. Decide whether each of the following statements is true or false.
   (*a*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   (*b*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   (*c*) $(A \cup B') \cap C' = (A \cup C') \cap B'$

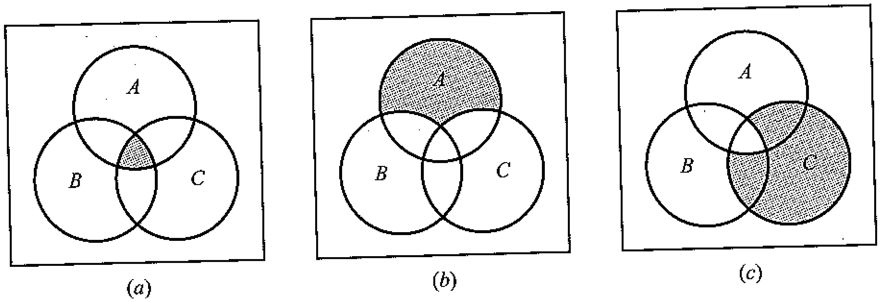


FIGURE 1.6

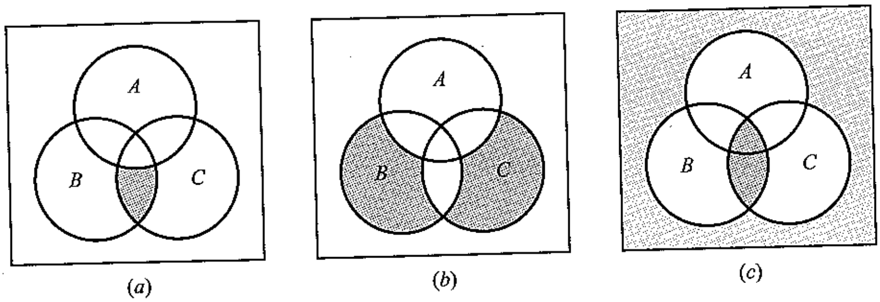
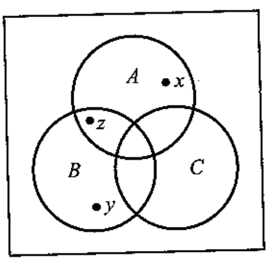


FIGURE 1.7

FIGURE 1.8



9. Determine which (if any) of the following set relations are true for *all* sets A, B, and C. (*Hint*: Use Venn diagrams.)
   (*a*) $A' \cap B' = (A \cap B)'$ (*b*) $(A \cap B') \subset A'$
   (*c*) $A' \cap B' \subset (A \cup B)'$ (*d*) $(A \cap B)' \subset A'$
   (*e*) $(A \cup B)' \cap C = (A' \cap B') \cup C$ (*f*) $(A \cap B)' \cup C' = (A' \cup B') \cup C'$
10. Which of the following is a true statement about the Venn diagram shown in Figure 1.8?
   (*a*) $x \in A \cap C$ (*b*) $z \in (A \cap B)' \cup C$ (*c*) $y \in B'$
   (*d*) $x \in C'$ (*e*) $y \in A' \cup C$
11. Let U be a universal set with disjoint subsets A and B; $n(U) = 60$, $n(A) = 25$, and $n(B) = 30$. Find $n((A \cup B)')$.
12. Let U be a universal set with disjoint subsets A and B; $n(U) = 55$, $n(A) = 25$, and $n(B) = 10$. Find $n(A' \cup B)$.
13. Let U be a universal set with disjoint subsets A and B; $n(A) = 25$, $n(A') = 40$, and $n(B') = 30$. Find $n(A \cup B)$.
14. Let $n(X \times Y) = 24$, $n(X \times Z) = 15$, and $n(Y \times Z) = 40$. Find $n(X \times Y \times Z)$.
15. Let X, A, B, and C be defined by

$$X = \{a, b, c, 1, 2, 3\}$$
$$A = \{a, b, c\} \qquad B = \{a, 2, 3\} \qquad C = \{1, 2, 3\}$$

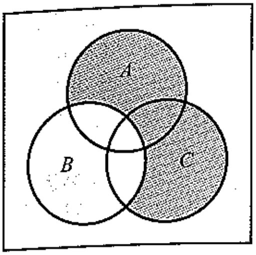


FIGURE 1.9

Which of the following pairs of subsets form a partition of X?

(a) A and B (b) A and C
(c) B and C (d) $(A \cup B)$ and $(C \cap B')$

16. The shaded region in Figure 1.9 is properly described by which two of the following:
(a) $(A \cup C) \cap B'$ (b) $(A \cup C) \cap (B' \cup C)$
(c) $(B' \cap C) \cup (B' \cap A)$ (d) $(A \cup B' \cup C)$

17. Let $A = \{1, 2, 3\}$ and $B = \{v, w\}$. By listing the elements in each of the three sets, show that $A \times \{v\}$ and $A \times \{w\}$ provide a partition of $A \times B$.

18. A set P is partitioned into subsets P_1, P_2, P_3. The number of elements in P_2 is 5 times the number in P_1 and the number of elements in P_3 is twice the number in P_1. If $n(P) = 40$, find $n(P_2)$.

19. Let $A = \{1, 7, 3, a, b\}$ and $B = \{3, c\}$. Determine
(a) $n(A \times B)$ (b) $n(B \times B \times B)$

20. A set U with $n(U) = 8$ is partitioned into 4 nonempty subsets A, B, C, and D. If all four sets are pairwise disjoint, then which of the following statements *must* be true?
(a) $n(A) + n(B) = n(C) + n(D)$ (b) $n(A) + n(B) \neq n(C) + n(D)$
(c) $n(A) + n(B) \geq 2$ (d) $n(A) + n(B) + n(C) + n(D) \neq 8$

Data surveys

21. A sociologist has a project which involves the collection of data. She is interested in data which can be obtained by mail, by phone, or in person in any of the cities of Atlanta, Boston, Chicago, Denver, or Elmira. She has funds for one project, that is, to collect data in one way from one city. Determine the number of possible ways to carry out the project.

22. Let A, B, and C be subsets of a universal set U with A and B disjoint, $n(U) = 110$, $n(A) = 35$, $n(B) = 44$, $n(A \cup B \cup C) = 96$, and $n((A \cup B) \cap C) = 28$. Find $n(C)$.

23. Let A, B, and C be distinct subsets of a universal set U with $A \subset B \subset C$. Also suppose $n(A) = 3$ and $n(C) = 7$. In how many different ways can you select B such that $n(B) = 4$? Repeat this exercise with $n(B) = 5$.

24. Suppose $n(A) = 5$, $n(B) = 10$, and $n(C) = 20$. Which of the following sets has more elements, $A \times B \times C$ or $B \times B \times B$?

25. A bag contains 6 green balls, 1 yellow ball, and 2 red balls. An experiment consists of selecting three balls, one after another without replacement, and noting the color of each ball selected. Suppose the set X of outcomes is partitioned into X_1, X_2, and X_3 where X_1 is the set of outcomes containing no red balls, X_2 is the set of outcomes containing 1 red ball, and X_3 is the set of outcomes containing 2 red balls. Find $n(X_1)$, $n(X_2)$, and $n(X_3)$.

26. Let A, B, and C be subsets of a universal set U with A and B disjoint, $n(U) = 120$, $n(A) = 35$, $n(B) = 44$, $n(A \cup B \cup C) = 100$, and $n((A \cup B) \cap C) = 28$. Find $n(C')$.

27. A set X is partitioned into subsets X_1, X_2, and X_3. The number of elements in X_1 is twice the number in X_2, and the number in X_3 is 5 times the number in X_2. If $n(X) = 40$, find $n(X_1)$, $n(X_2)$, and $n(X_3)$.

28. A set X with $n(X) = 45$ is partitioned into three subsets X_1, X_2, and X_3. If $n(X_2) = 2n(X_1)$ and $n(X_3) = 3n(X_2)$, find the number of elements in subset X_1.

29. A set X with $n(X) = 60$ is partitioned into subsets $X_1, \ldots, X_6$. If $n(X_1) = n(X_2) = n(X_3)$, $n(X_4) = n(X_5) = n(X_6)$ and $n(X_1) = 4n(X_4)$, find $n(X_1)$.

30. Let A_1 and A_2 be a partition of A, and let B_1 and B_2 be a partition of B. Is it true that $A_1 \times B_1$, $A_1 \times B_2$, $A_2 \times B_1$, and $A_2 \times B_2$ form a partition of $A \times B$? Why or why not?

31. A set X with $n(X) = 100$ is partitioned into subsets A, B, C, D, and E. Suppose $n(B) = 3n(A)$, $n(C) = 4n(A)$, $n(E) = n(B) + n(C)$, and $n(D) = n(E) - 10$. Find the number of elements in subset D.