

3. Find all real solutions of

$$\frac{1}{x-1} + \frac{1}{x+2} = \frac{5}{4}$$

Observe that $x \neq 1, x \neq -2$.

Use a common denominator to add the fractions on the left hand side (LHS):

$$\frac{(x+2) + (x-1)}{(x-1)(x+2)} = \frac{5}{4}$$

Simplify the numerator.

$$\frac{2x+1}{(x-1)(x+2)} = \frac{5}{4}$$

Cross-multiply to eliminate the denominator:

$$4(2x+1) = 5(x-1)(x+2)$$

Expand both sides.

$$8x+4 = 5(x^2+x-2)$$

That is,

$$8x+4 = 5x^2+5x-10$$

Combine like terms to get this in the form

(polynomial = 0):

$$0 = 5x^2 - 3x - 14$$

Factor the polynomial:

$$0 = (5x+7)(x-2)$$

The product of two numbers may be zero only if one of the two numbers is zero.

So

$$0 = 5x+7 \text{ or } 0 = x-2$$

That is,

$$x = -\frac{7}{5} \text{ or } x = 2$$

One may also find the roots of a quadratic by completing the square or by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

In our case,

$$x = \frac{3 \pm \sqrt{9 + 4 \cdot 5 \cdot 14}}{2 \cdot 5}$$

$$= \frac{3 \pm \sqrt{289}}{10}$$

$$= \frac{3 \pm 17}{10}$$

$$= \left\{ \frac{20}{10} = 2 \text{ or } \frac{-14}{10} = -\frac{7}{5} \right\}$$

as before.

5x	-2	-10x
x	7	7x
5x ²	-14	-3x

Name: E. Alec Johnson

Give an answer. No calculators are allowed.

1. Find the equation of the line going through points $(-1, \frac{2}{3})$ and $(\frac{-2}{3}, -2)$. What are its x and y intercepts?

$$\begin{aligned} \text{Slope} = m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(-2) - (\frac{2}{3})}{(\frac{-2}{3}) - (-1)} \\ &= \frac{(-\frac{8}{3})}{(\frac{1}{3})} \end{aligned}$$

$$m = -8$$

$$\begin{aligned} y &= mx + b \\ \frac{2}{3} &= (-8)(-1) + b \\ \frac{2}{3} &= 8 + b \\ b &= \frac{2}{3} - 8 \\ &= \frac{2}{3} - \frac{24}{3} \end{aligned}$$

$$b = -\frac{22}{3}$$

$$y = -8x - \frac{22}{3}$$

y-intercept

Set $x = 0$.

$$\text{Get } y = -\frac{22}{3}$$

x-intercept

Set $y = 0$.

$$0 = -8x - \frac{22}{3}$$

$$8x = -\frac{22}{3}$$

$$x = -\frac{22}{3} \cdot \frac{1}{8}$$

$$x = -\frac{11}{12}$$

2. Draw the graphs of $y - 3x = -2$ and $-2y + 5x = 7$. Find the coordinates of the intersection point.

Intersection

$$\begin{cases} y - 3x = -2 \\ -2y + 5x = 7 \end{cases}$$

multiply the first equation by 2 and add the second equation:

$$\begin{array}{r} 2y - 6x = -4 \\ -2y + 5x = 7 \\ \hline -x = 3 \\ \hline x = -3 \end{array}$$

To find y plug this value of x into one of the equations and solve for y :

$$\begin{aligned} y - 3(-3) &= -2 \\ y + 9 &= -2 \end{aligned}$$

$$y = -11$$

The graph should show intercepts.

Equation	y-intercept	x-intercept
$y - 3x = -2$	-2	$\frac{2}{3}$
$-2y + 5x = 7$	$-\frac{7}{2}$	$\frac{7}{5}$

