Practice exam for Math 234 Midterm II (A. Assadi, Fall 2009).

Problem 1. Evaluate the iterated integral

$$\int_{-1}^{1} dx \int_{-x}^{x^2} (3+2y)dy.$$

Problem 2. Expand the function $f(x,y) = y^2 e^{x+y} + x^3 y^2 + 5$ in a Taylor series centered around the origin $(0,0)$ out to fourth order.

Problem 3. The plane $x+y+z=2$ cuts the cylinder $x^2+y^2=4$ in an ellipse. Find the points on the ellipse that lie closest to and farthest from the origin.

Problem 4.

1. Find the Jacobian of the transformation $x = u$, $y = uv$ and sketch the region $G: 1 \le u \le 2$, $1 \le uv \le 2$ in the uv-plane.
2. Transform the integral

$$\int_1^2 \int_1^2 \frac{y}{x}\, dy\, dx$$

into an integral over G, and evaluate both integrals.

Problem 5. A thin plate covers the triangular region bounded by the x-axis and the lines $x=2$ and $y=2x$ in the first quadrant. The plate's density at the point (x,y) is $\delta(x,y) = x+2y+1$. Find the plate's mass and its center of mass.

Problem 6. Let R be the region in the first quadrant bounded by the coordinate axes and the curves $y = 4-x$ and $x=2$.

1. Sketch the region R and set up the integral of the function $f(x,y) = xy$ over the region R with dx on the outside.
2. Reverse the order of integration in the above integral.
3. Compute the integral.

Problem 7. A cone is drilled (vertically) out of a ball of radius a so that the point of the cone is at the center of the ball and the hole has diameter a where it intersects the surface of the ball. Compute the volume of the solid that remains.

Problem 8. The container V is defined to be the volume of the spherical shell of inner radius 2 and outer radius 3. The container V is placed on the plane $z=-3$, and it is filled with a substance that has density *proportional* to *depth* of the substance from the plane. (Hint: the substance has the largest density near the bottom, and its density is smaller near the top. If we call the fixed constant of proportionality c, then the density equation is given by $M = c(z+3)$.)

1. Find the mass of the substance in V in terms of c.
2. Find the mass of the substance if the container is filled up to the *height* equal to three-quarters of its outer radius (the bottom starts from the plane $z=-3$ and the height of the portion within a quarter radius of the top is empty).