MATH 234 (A. Assadi) - Spring 2008 - MIDTERM I

Thursday, March 6, 2008

NAME (print): ______________________________

Instructions:

1. Write your name on each page.
2. Circle the name of your TA:

WU TONEJC

3. Closed book. Closed notes. Calculators allowed. No laptops.
4. 2 index cards of size $3'' \times 5''$ with formulas allowed.
5. Answer your questions on the exam paper. There are some extra pages in the back if you need them.
6. Show all your work. Partial credit is given only if your work is clear.
7. Enclose your final answers clearly in a rectangle.
8. Time allowed: 70 minutes for your work, plus 5 minutes for checking your answers.

Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	
Problem 6	20	
Problem 7	20	
Total	140	

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Problem 1. A particle is moving downwards along the helix

$$\mathbf{r}(t) = 3\cos(2t)\mathbf{i} + 3\sin(2t)\mathbf{j} - t\mathbf{k}.$$

(a) Compute the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle.

(b) How far does the particle travel along its path from $t = 0$ to $t = 2\pi$?

(c) Find the unit tangent vector $\mathbf{T}$ and the unit normal vector $\mathbf{N}$ and compute the angle between $\mathbf{T}$ and $\mathbf{N}$.

(d) Compute the curvature κ and the radius of the curvature. What can you say about κ?

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Problem 2. Find and graph the osculating circle (i.e. the circle of curvature) for the parabola

$$P : x = 1 - y^2$$

at the point of intersection of P with the x-axis.

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Problem 3. If capacitors of C_1, C_2 and C_3 microfarads are connected in series to make a C-microfarad capacitor, the value of C can be found from the equation

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Find the value of $\frac{\partial C}{\partial C_1}$ when $C_1 = 8$, $C_2 = 12$ and $C_3 = 24$ microfarads.

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Problem 4. Find the extreme values of $f(x, y) = x^2 + 3y^2 + 2y$ on the unit circle $x^2 + y^2 = 1$.

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Problem 5. Find the equation of the tangent plane and the normal line for the surface S given by

$$x^2 - 3y^2 - 25z^2 = -11$$

at the point $\mathbf{p} = (1, -2, 0)$.

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Problem 6. Find all critical points for

$$f(x, y) = 2x^4 - 4x^2 + y^2 + 2y$$

and indicate whether each point is a local maximum, local minimum or a saddle point.

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Problem 7. Let

$$f(x, y) = x^2 + y^2 - 4.$$

(a) Sketch the level sets $f(x, y) = k$ for $k = -4$, -3, 0 and 5.

(b) Sketch the graph of $z = f(x, y)$.

(c) Find a unit vector $\mathbf{v}$, in the direction in which $f(x, y)$ *decreases* most rapidly at the point $\mathbf{p} = (1, -2\sqrt{2})$. What is the rate of change in this direction?

(d) Sketch the vector $\mathbf{v}$ at the point $\mathbf{p}$ on the graph in part (a). What do you observe?

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SCRATCH WORK

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SCRATCH WORK