

5.3 # 6, 9, 12, 16, 23, 26, 33, 40, 41

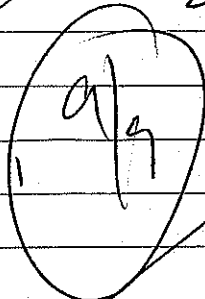
(8)  $y'' + 5y' + 5y = 0$   
 $r^2 + 5r + 5 = 0$      $r = \frac{-5 \pm \sqrt{25 - 20}}{2} = -\frac{5}{2} \pm \frac{\sqrt{5}}{2}$

$y(x) = c_1 e^{(-\frac{5+\sqrt{5}}{2})x} + c_2 e^{(-\frac{5-\sqrt{5}}{2})x}$

(9)  $y'' + 8y' + 25y = 0$   
 $r^2 + 8r + 25 = 0$      $r = \frac{-8 \pm \sqrt{64 - 100}}{2} = -4 \pm \sqrt{36} = -4 \pm 3i$

$y(x) = c_1 e^{-4x} \cos 3x + c_2 e^{-4x} \sin 3x$

(12)  $y^{(4)} - 3y^{(3)} + 3y'' - y' = 0$     ROOTS  
 $r^4 - 3r^3 + 3r^2 - r = 0$     0, 1, 1, 1  
 $r(r^3 - 3r^2 + 3r - 1) = 0$   
 $r(r-1)^3 = 0$



$y(x) = c_1 + c_2 e^x + c_3 x e^x + c_4 x^2 e^x$

(16)  $y^{(4)} + 18y'' + 81y = 0$   
 $r^4 + 18r^2 + 81 = 0$   
 $(r^2 + 9)^2 = 0$   
 $r^2 + 9 = 0$   
 $r = 3i$

$y(x) = c_1 \cos 3x + c_2 \sin 3x + c_3 x \cos 3x + c_4 x \sin 3x$   
 $= (c_1 + c_3 x) \cos 3x + (c_2 + c_4 x) \sin 3x$

(23)  $y'' - 6y' + 25y = 0$ ;  $y(0) = 3$ ,  $y'(0) = 1$   
 $r^2 - 6r + 25 = 0$      $r = \frac{6 \pm \sqrt{36 - 100}}{2} = 3 \pm 4i$

$y(x) = c_1 e^{3x} \cos 4x + c_2 e^{3x} \sin 4x$

$\rightarrow y(0) = 3 = c_1$

$y'(x) = 3c_1 e^{3x} \cos 4x - 4c_1 e^{3x} \sin 4x + 3c_2 e^{3x} \sin 4x + 4c_2 e^{3x} \cos 4x$

$y'(0) = 1 = 3c_1 + 4c_2$ ,  $c_1 = 3$

$1 = 9 + 4c_2$

$-8 = 4c_2$

$c_2 = -2$

$y(x) = 3e^{3x} \cos 4x - 2e^{3x} \sin 4x$

26)  $y^{(3)} + 10y'' + 25y' = 0; y(0) = 3, y'(0) = 4, y''(0) = 5$

$$r^3 + 10r^2 + 25r = 0$$

$$r(r^2 + 10r + 25) = 0$$

$$r = \frac{-10 \pm \sqrt{100 - 100}}{2} = \frac{-10}{2} = -5 \quad \text{roots } 0, -5, -5$$

$$\Rightarrow y(x) = c_1 + (c_2 + c_3 x)e^{-5x} = c_1 + c_2 e^{-5x} + c_3 x e^{-5x}$$

$$y(0) = 3 = c_1 + c_2$$

$$\Rightarrow y'(x) = -5c_2 e^{-5x} - 5c_3 x e^{-5x} + c_3 e^{-5x}$$

$$y'(0) = 4 = -5c_2 + c_3$$

$$\Rightarrow y''(x) = 25c_2 e^{-5x} + 25c_3 x e^{-5x} - 5c_3 e^{-5x} + -5c_3 e^{-5x}$$

$$y''(0) = 5 = 25c_2 - 10c_3$$

$$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & -5 & 1 & 4 \\ 0 & 25 & -10 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & -5 & 1 & 4 \\ 0 & 0 & -5 & 5 \end{bmatrix}$$

$$-5c_3 = 25$$

$$c_3 = 5$$

$$-5c_2 + c_3 = 4$$

$$-5c_2 = 4 - 5$$

$$c_2 = \frac{1}{5}$$

$$c_1 + c_2 = 3$$

$$c_1 = 3 - \frac{1}{5}$$

$$c_1 = \frac{24}{5}$$

$$y(x) = \frac{24}{5} + \frac{1}{5} e^{-5x} + 5x e^{-5x} = \frac{1}{5} (24 + e^{-5x} + 25x e^{-5x})$$

27)  $y^{(3)} + 3y'' - 5y' = 0; y = e^{3x}$

$$r^3 + 3r^2 - 5r = 0$$

$$r-3 \mid r^3 + 3r^2 - 0 - 5r$$

$$-(r^3 - 3r^2)$$

$$(6r^2 - 0)$$

$$-(6r^2 + 18r)$$

$$+18r - 5r$$

$$-(18r - 5r)$$

$$0$$

$$(r-3)(r^2 + 6r + 18) \rightarrow r = \frac{-6 \pm \sqrt{36 - 72}}{2} = -3 \pm 3i$$

$$y(x) = c_1 e^{3x} + c_2 e^{-3x} \cos 3x + c_3 e^{-3x} \sin 3x$$

$$(40) y(x) = Ae^{2x} + B\cos 2x + C\sin 2x$$

solution  
for root  
 $r=2$   
 $(r-2)=0$   
↓

$$B e^{0x} \cos 2x + C e^{0x} \sin 2x$$

⇒ solution for root  $r = 0 \pm 2i$

$$(2i)^2 = r^2$$

$$r - 2i = 0$$

$$\leftarrow -4 = r^2$$

$$r^2 + 4 = 0$$

characteristic equation:

$$(r-2)(r^2+4)=0$$

$$r^3 + 4r - 2r^2 - 8 = 0$$

$$r^3 - 2r^2 + 4r - 8 = 0$$

↓

differential equation:

$$y^{(3)} - 2y'' + 4y' - 8y = 0$$

$$(41) y(x) = A\cos 2x + B\sin 2x + C\cosh 2x + D\sinh 2x$$

solution for root:

$$r^2 = 0 \pm 2i^2$$

$$r^2 = -4$$

$$r^2 + 4 = 0$$

solution for root

$$r^2 = \pm 2^2$$

$$r^2 = 4$$

$$r^2 - 4 = 0$$

characteristic equation:

$$(r^2+4)(r^2-4)=0$$

$$r^4 + 4r^2 - 4r^2 - 16 = 0$$

$$r^4 - 16 = 0$$

↓

differential equation:

$$y^{(4)} - 16y = 0$$