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Section 324
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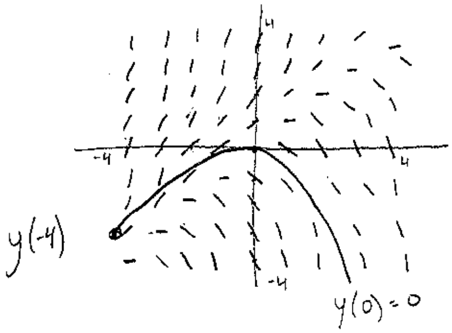
HW 2

1.3 - 22
1.4 - 3 12 22 48
1.5 - 8 14 16 31 32 37

$\boxed{22}$ $y' = y - x$



$y(-4) = -3$

$\boxed{3}$ $\frac{dy}{dx} = y \sin x$

$\int \frac{dy}{y} = \int \sin x \, dx$

$\ln|y| = -\cos x + c$

$y = \pm C e^{-\cos x}$

$\boxed{12}$ $y \frac{dy}{dx} = x(y^2+1)$

$\int \frac{y}{y^2+1} dy = \int x \, dx$

$u = y^2+1 \quad du = 2y\,dy$

$\frac{1}{2}\int \frac{du}{u} = \frac{1}{2}x^2 + c$

$\frac{1}{2}\ln|u| = \frac{1}{2}x^2 + c$

$\ln\sqrt{y^2+1} = \frac{1}{2}x^2 + c$

$\sqrt{y^2+1} = Ce^{\frac{1}{2}x^2}$

$y^2 + 1 = Ce^{x^2}$

$\boxed{22}$ $\frac{dy}{dx} = 4x^3y - y \quad y(1) = 3$

$\frac{dy}{dx} = y(4x^3 - 1)$

$\int \frac{dy}{y} = \int (4x^3 - 1)\, dx$

$\ln|y| = x^4 - x + c$

$y = \pm e^{x^4 - x + c}$

$y = Ce^{x^4 - x}$

$3 = C$

$y = 3e^{x^4 - x}$

$\boxed{48}$ ${}^{238}U = 4.51 \times 10^9$ yrs

${}^{235}U = 7.10 \times 10^8$ yrs

current $\frac{{}^{238}U}{{}^{235}U} = 137.7 \qquad \frac{{}^{238}U_0}{{}^{235}U_0} = 1$

$\div \quad \frac{{}^{238}U = U_0 e^{-K_1 t}}{{}^{235}U = U_0 e^{-K_2 t}} \qquad K = \frac{\ln 2}{T_{1/2}}$

$137.7 = e^{-K_1 t + K_2 t}$

$137.7 = e^{t(-K_1 + K_2)} \qquad K_1 = \frac{\ln 2}{4.51 \times 10^9} \qquad K_2 = \frac{\ln 2}{7.10 \times 10^8}$

$137.7 = e^{(8.226 \times 10^{-10})t} \qquad -K_1 + K_2 = 8.226 \times 10^{-10}$

$\frac{\ln 137.7}{8.226 \times 10^{-10}} = t$

$t = 5.987 \times 10^9$

$\boxed{8}$ $3xy' + y = 12x$

$$y' + \frac{1}{3x} y = 4$$

$$y' + \frac{1}{3x} y = 4$$

$$\rho = e^{\frac{1}{3}\int \frac{1}{x} dx} = e^{\frac{1}{3}\ln x + c} = \sqrt[3]{x}$$

$$y\sqrt[3]{x} = \int 4\sqrt[3]{x}$$

$$y x^{1/3} = 3x^{4/3} + C$$

$$y = 3x + \frac{C}{\sqrt[3]{x}}$$

$\boxed{14}$ $xy' - 3y = x^3 \qquad y(1) = 10$

$$y' - \frac{3}{x} y = x^2$$

$$\rho = e^{-3\int \frac{1}{x} dx} = e^{-3\ln x} = \frac{1}{x^3}$$

$$\frac{y}{x^3} = \int x^2 \cdot \frac{1}{x^3}\, dx$$

$$\frac{y}{x^3} = \ln x + C$$

$$y = x^3 \ln x + Cx^3$$

$$10 = 0 + C(1)^3$$

$$C = 10$$

$$y = x^3(\ln x + 10)$$

$\boxed{16}$ $y' = (1-y)\cos x \qquad y(\pi) = 2$

$$\frac{dy}{dx} = (1-y)\cos x$$

$$\int \frac{dy}{1-y} = \int \cos x\, dx$$

$$-\ln|1-y| = \sin x + c$$

$$e^{-\ln|1-y|} = Ce^{\sin x}$$

$$\frac{1}{1-y} = Ce^{\sin x}$$

$$y = \frac{1}{Ce^{\sin x}} + 1$$

$$2 = \frac{1}{Ce^{0}} + 1$$

$$C = 1$$

$$y = e^{-\sin x} + 1$$

$\boxed{31}$ $y_c(x) = Ce^{-\int P(x)}dx$ solves $y' + Py = 0$

(a) $y_c' = -PCe^{\int P(x)}$

$$-PCe^{\int P} + P\left[Ce^{-\int P}\right] = 0 \quad \checkmark$$

(b) $y_p = e^{-\int P(x)}\left[\int Q(x)e^{\int P(x)}dx\right]$

$$y_p' = \left[-P(x)e^{-\int P(x)}\right]\left[\int Q(x)e^{\int P(x)}\right] + \left[e^{-\int P(x)}Q(x)e^{\int P(x)}\right]$$

$$\Rightarrow y_p' + P\left[e^{-\int P(x)}\left[\int Q(x)e^{\int P(x)}\right]\right] = Q(x)e^{-\int P}e^{\int P}$$

$$y_p' + Py_p = Q(x)$$

(c) $y_c + y_p = Ce^{-\int P} + \left(e^{-\int P}\right)\left(\int Qe^{\int P}\right) = y(x)$

$$y'(x) = -PCe^{\int P} + \left(-Pe^{-\int P}\right)\left(\int Qe^{\int P}\right) + \left(e^{-\int P}Qe^{\int P}\right)$$

$$-PCe^{\int P} - Pe^{-\int P}\int Qe^{\int P} + e^{-\int P}Qe^{\int P} + P\left[Ce^{-\int P} + e^{-\int P}\left(\int Qe^{\int P}\right)\right] = Q$$

$$e^{-\int P}\cdot Q\cdot e^{\int P} = Q$$

$$Q = Q$$

$\boxed{32}$ $y' + y = 2\sin x$

(a) $y_p = A\sin x + B\cos x$

$y_p' = A\cos x - B\sin x$

$A\cos x - B\sin x + A\sin x + B\cos x = 2\sin x$

$(A+B)\cos x + (A-B)\sin x = 2\sin x$

$A = 1 \quad B = -1$

$y_p = \sin x - \cos x$

(b) $y_h = Ce^{-x}$

(c) $y(0) = 1$

$y = Ce^{-x} + \sin x - \cos x$

$1 = C + (-1)$

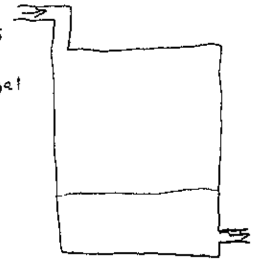
$C = 2$

$y = 2e^{-x} + \sin x - \cos x$

37

$r_i = 5 \text{ gal/s}$

$c_i = 1 \text{ lb/gal}$



$V_0 = 100 \text{ gal}$

$x_0 = 50 \text{ lbs}$

$r_0 = 3 \text{ gal/s}$

$$V(t) = 100 + 2t$$

$$C_0 = \frac{x(t)}{V(t)}$$

$$\frac{dx}{dt} = r_i c_i - r_o c_o$$

$$\frac{dx}{dt} = r_i c_i - \frac{r_o}{V} x$$

$$\frac{dx}{dt} = 5 - \frac{3}{100 + 2t} x$$

$$\frac{dx}{dt} + \frac{3}{100+2t} x = 5$$

$$\rho = e^{\int \frac{3}{100+2t}} = e^{\frac{3}{2}\ln(t+50)} = (t+50)^{3/2}$$

$$x(t+50)^{3/2} = 5\int (t+50)^{3/2}$$

$$x(t+50)^{3/2} = 2(t+50)^{5/2} + c$$

$$x = 2(t+50) + \frac{c}{(t+50)^{3/2}}$$

$$x(0) = 50$$

$$50 = 2(50) + \frac{c}{50^{3/2}}$$

$$-50 = \frac{c}{50^{3/2}}$$

$$c = -50^{5/2}$$

$$* \quad x = 2t + 100 - \frac{50^{5/2}}{(t+50)^{3/2}}$$

$$400 = 100 + 2t$$

$$t = 150 \text{ s}$$

$$* \quad x(150) = 393.75 \text{ lbs salt}$$