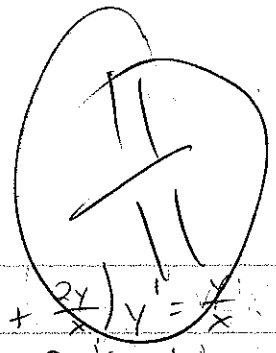


Codi Klapperich
Math 320 HW#3
1.6 6, 10, 18, 22, 34, 36, 39
2.1 5, 6, 21, 28

6. $(x+2y)y' = y$ $\quad (1+\frac{2y}{x})y' = \frac{y}{x}$ $\quad v = \frac{y}{x}$ $\quad y = vx$

$(1+2v)(v+x\frac{dv}{dx}) = v$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = \frac{v}{1+2v}$ $\quad x\frac{dv}{dx} = \frac{v-v(1+2v)}{1+2v} = \frac{-2v^2}{1+2v}$

$\frac{1+2v}{v^2}dv = -\frac{2}{x}dx$ $\quad \left(\frac{1}{v^2}+\frac{2}{v}\right)dv = -\frac{2}{x}dx$ $\quad -\frac{1}{v} + 2\ln v = -2\ln x + C$

$y\left[-\frac{x}{y} + 2\ln\frac{y}{x} = -2\ln x + C\right]y$ $\quad \boxed{2y\ln y = x + Cy}$

10. $xyy' = x^2 + 3y^2$ $\quad v = \frac{y}{x}$ $\quad vx = y$ $\quad \frac{dy}{dx} = v + x\frac{dv}{dx}$

$y' = \frac{x}{y} + \frac{3y}{x}$

$v + x\frac{dv}{dx} = \frac{1}{v} + 3v = \frac{1}{v} + 2v$ $\quad \frac{1+2v^2}{v}$

$\frac{1}{4}\int \frac{v\,dv}{1+2v^2} = \int\frac{dx}{x}$ $\quad u = 1+2v^2$, $du = 4v\,dv$

$\frac{1}{4}\ln(1+2v^2) = \ln(x+C)$ $\quad (1+2v^2)^{1/4} = Cx$

$(1+2\frac{y^2}{x^2})^{1/4} = Cx$

$1 + 2\frac{y^2}{x^2} = C'x^4$

$\boxed{x^2 + 2y^2 = C''x^6}$

18. $(x+y)y' = 1$ $\quad z = x+y$ $\quad \frac{dz}{dx} = 1 + \frac{dy}{dx}$

$\frac{dz}{dx} - 1 = \frac{1}{z}$ $\quad \frac{dy}{dx} = \frac{dz}{dx} - 1$

$\frac{dz}{dx} = \frac{1}{z} + 1$

$\int\frac{dz}{\frac{1}{z}+1} = \int dx = z - \ln|z+1| = x + C$

$x+y - \ln|x+y+1| = x + C$

$\boxed{y = \ln|x+y+1| + C}$

22. $x^2y' + 2xy = 5y^4$ $\quad y' + \frac{2}{x}y = \frac{5}{x^2}y^4$ $\quad v = y^{1-n} = y^{-3}$ $\quad v = \frac{1}{y^3}$ $\quad y = \sqrt[-3]{\frac{1}{v}}$

$\frac{dv}{dx} - 3\left(\frac{2}{x}\right)v = -3\cdot\frac{5}{x^2}$

$\rho = e^{-\int 6\frac{1}{x}dx} = e^{-6\ln x} = x^{-6}$ $\quad \frac{d}{dx}\left(vx^{-6}\right)' = \frac{-15}{x^2}\cdot x^{-6}$ $\quad \int\frac{-15}{x^7}$ $\quad \frac{+15x^{-7}}{+7}$

$vx^{-6} = \frac{15}{7x^7}\cdot x^6$

$v = \frac{15}{7x} + Cx^6$ $\quad \frac{15+7Cx^7}{7x}$

$y^{-3} = \frac{15+7Cx^7}{7x}$

$\boxed{y^3 = \frac{7x}{15+7Cx^7}}$

34 $(2xy^2+3x^2)dx+(2x^2y+4y^3)dy=0$

$M_y=4xy \qquad N_x=4xy$

$F_x=M \quad x^2y^2+x^3+g(y) \qquad F_y=N \quad x^2y^2+x^4$

$\boxed{x^3+x^2y^2+y^4=C}$

36. $(1+ye^{xy})dx+(2y+xe^{xy})dy=0$

$M_y=xye^{xy} \qquad N_x=xye^{xy}$

$F_x=M \quad x+e^{xy}+g(y) \qquad F_y=N \quad y^2+e^{xy}$

$\boxed{x+e^{xy}+y^2=C}$

37. $(\cos x+\ln y)dx+(\frac{x}{y}+e^y)dy=0$

$M_y=\frac{1}{y} \qquad N_x=\frac{1}{y}$

$F_x=M \qquad F_y=N$

$\sin x+g(y) \qquad x\ln y+e^y+C$

$\boxed{\sin x+x\ln y+e^y=C}$

5 $\frac{dx}{dt} = 3x(5-x)$ $x(0)=8$

$\int \frac{dx}{x(5-x)} = \int 3\,dt$ $\frac{1}{x(5-x)} = \frac{A}{x} + \frac{B}{5-x}$ $A(5-x)+Bx$

$\frac{1}{5}\int \frac{dx}{x} + \frac{1}{5}\int \frac{dx}{5-x} = 5\int 3\,dt$ $5A - Ax + Bx$ $5A=1$ $-A+B=0$

$\ln x - \ln|5-x| = 15t + C$ $A=\frac{1}{5}$ $B=\frac{1}{5}$

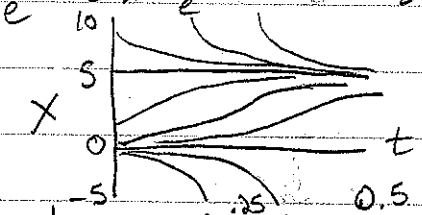
$\ln \frac{x}{5-x} = 15t + C$ $\frac{x}{5-x} = Ce^{15t} = f$ $\frac{x}{5-x} = f$ $x = (5-x)f$ $5f - xf$ $x(1+f)$ $x + xf = 5f$

$x = \frac{5Ce^{15t}}{1+Ce^{15t}}$ $x = \frac{5f}{1+f}$

$\frac{-\frac{40}{3}e^{15t}/e^{15t}}{1-\frac{8}{3}e^{15t}/e^{15t}} \cdot (10) = 8 = \frac{5C\cdot 1}{1+C\cdot 1} = \frac{5C}{1+C}$ $8+8C=5C$

$\frac{-40/3}{e^{-15t}-8/3} \cdot x - 3 = \boxed{\frac{40}{8-3e^{-15t}}}$ $8=-3C$

$-\frac{8}{3} = C$



6. $\frac{dx}{dt} = 3x(x-5)$ $x(0)=2$

$\int \frac{dx}{x(x-5)} = \int 3\,dt$ $\frac{1}{x(x-5)} = \frac{A}{x} + \frac{B}{x-5}$ $\frac{1}{5}\int -\frac{1}{x} + \frac{1}{x-5} = 5\int 3t$

$1 = A(x-5) + Bx$ $-\ln|x| + \ln|x-5| = 15t + C$

$1 = A(-5) + (A+B)x$ $e^{\ln\left|\frac{x-5}{x}\right|} = e^{15t+C}$

$1 = A(-5)$ $0 = A+B$ $\frac{x-5}{x} = Ce^{15t} = f$

$A = -\frac{1}{5}$ $B = \frac{1}{5}$ $x - 5 = xf$

$x(1-f) = 5$

$x = \frac{5}{1-f} = \frac{5}{1-Ce^{15t}}$

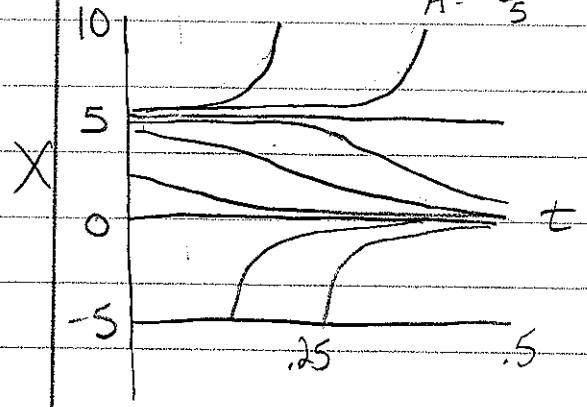
$x(0) = 2 = \frac{5}{1-Ce^{0}} = \frac{5}{1-C}$

$1 - C = \frac{5}{2}$

$C = -\frac{3}{2}$

$x = \frac{5}{1+\frac{3}{2}e^{15t}} \cdot 2$

$\boxed{x = \frac{10}{2+3e^{15t}}}$



21. $\frac{dP}{dt} = kP(200-P)$ k=const 1940 100 million grow at 1 mil/yr

$1 = K\,100(200-100)$ $k = 1\times10^{-4}$ Predict for 2000

$\int \frac{dP}{P(200-P)} = \int k\,dt$ $\frac{1}{P(200-P)} = \frac{A}{P} + \frac{B}{200-P}$

$\frac{1}{200}\int \frac{1}{P}dP + \frac{1}{200}\int \frac{1}{200-P}dP = \int k\,dt$ $A(200-P) + BP$

$\frac{1}{200}\ln P - \frac{1}{200}\ln|200-P| = kt + C$ $1 = A(200) - AP + BP = 0$

$\ln\frac{P}{200-P} = 200kt + C$ $\frac{1}{200} = A$ $-A + B = 0$

$e^{\ln\frac{P}{200-P}} = e^{200kt+C}$ $B = \frac{1}{200}$

$\frac{P}{200-P} = Ce^{200kt}$

$P = 200Ce^{200kt} - PCe^{200kt}$

$P(1+Ce^{200kt}) = 200Ce^{200kt}$

$P = \frac{200Ce^{200kt}}{1+Ce^{200kt}} \Big/ Ce^{200kt} = \frac{200}{Ce^{-200kt}+1}$ $100 = \frac{200}{C+1}$ $100C+100 = 200$

$\frac{200}{1+e^{-200(1\times10^{-4})(60)}}$ $100C = 100$

$C = 1$

$P(60) = \frac{200}{1+e^{-.6}} = 153.7$ million

28. $\frac{dx}{dt} = 0.0001x^2 - 0.01x$ $x(0) = 25$ $\frac{1}{0.0001x(x-100)} = \frac{A}{0.0001x} + \frac{B}{x-100}$

$\frac{dx}{0.0001x^2 - 0.01x} = dt$ $\frac{dx}{0.0001x(x-100)}$ $Ax - A100 + B\cdot0.0001x$

$-100\int\frac{1}{x}dx + 100\int\frac{1}{x-100}dx = \int dt$ $1 = -100A$ $A = -.01$

$100\ln|x-100| - 100\ln|x| = t + C$ $0 = Ax + B\cdot0.0001x$

$\frac{1}{100}100\ln\frac{|x-100|}{|x|} = \frac{1}{100}t + C$ $A + .0001B = 0$

$\frac{x-100}{x} = Ce^{\frac{1}{100}t}$ $B = 100$

$x - 100 = xCe^{\frac{1}{100}t}$ $x(1-Ce^{\frac{1}{100}t}) = 100$

$x = \frac{100}{1-Ce^{\frac{1}{100}t}}$ $x(0) = 25 = \frac{100}{1-C}$ $25 - 25C = 100$

a) alligators will die off $x(t) = \frac{100}{1+3e^{\frac{1}{100}t}}$ $-25C = 75$

$x(100) = \frac{100}{1+3e^{1}}$ $C = -3$

b) alligators will die off later than part (a) $x(0) = 150 = \frac{100}{1-Ce^{\frac{1}{100}t}}$ $x(t) = \frac{100}{1-\frac{1}{3}e^{\frac{1}{100}t}}$

$150 = \frac{100}{1-C}$ $0 = \frac{100}{1-\frac{1}{3}e^{\frac{1}{100}t}}$

$150 - 150C = 100$

Doomsday occurs at about 9 yrs 2 months $-150C = -50$ $C = \frac{1}{3}$