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Math 320, Sec. 323
Homework #4

9.5/10

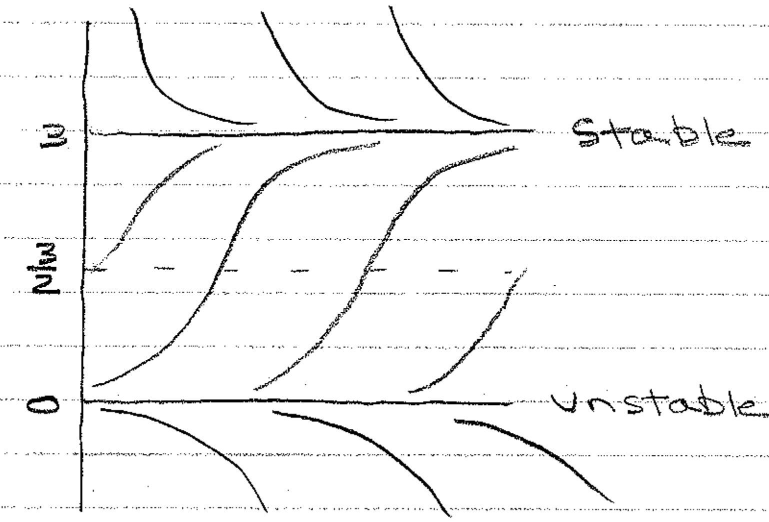
Slope Field w/ soln. Curves

2.2: 4, 9, 14, 21, 29 2.4: 4, 7 3.1: 4, 14, 23

4. $\frac{dx}{dt} = 3x - x^2$ $f(x) = dx/dt$ $f(x) = x(3-x)$

Critical Points: $x = 0, 3$

$x = 0$ $x = 3$
$f(1) = (+)$ $f(2) = +$
$f(-1) = (-)$ $f(4) = -$
unstable stable



$\frac{dx}{dt} = x(3-x)$

$\frac{dx}{x(3-x)} = dt$

Partial Fractions:

$1 = x(3-x)\left(\frac{A}{x} + \frac{B}{3-x}\right)$ $1 = 3A - xA + xB$

$A = \frac{1}{3}$ $0 = B - A$ $B = \frac{1}{3}$

$\int\left(\frac{1}{x} + \frac{1}{3-x}\right)dx = \int 3\,dt \longrightarrow \ln\left|\frac{x}{3-x}\right| = 3t + C$

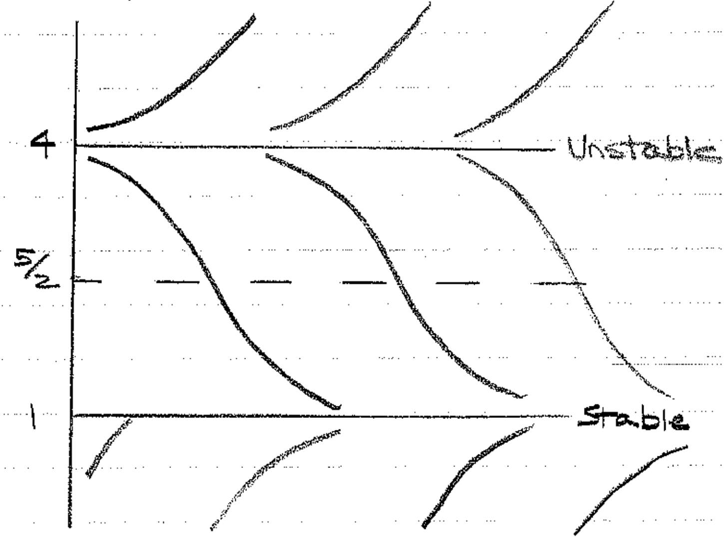
$\frac{x}{3-x} = Ce^{3t}$ $\frac{1}{Ce^{3t}} = \frac{3}{x} - 1$ $\frac{1}{Ce^{3t}} + 1 = \frac{3}{x}$

$x\left(\frac{1}{Ce^{3t}} + 1\right) = 3$ $\boxed{x = \frac{3Ce^{3t}}{1 + Ce^{3t}}}$

9. $\dfrac{dx}{dt} = x^2 - 5x + 4 \qquad f(x) = dx/dt \qquad f(x) = (x-1)(x-4)$

Critical Points: $x = 1, 4$

$x=1$	$x=4$
$f(2) = -$	$f(5) = +$
$f(0) = +$	$f(3) = -$
stable	unstable



$$\frac{dx}{(x-1)(x-4)} = dt$$

Partial Fractions:

$$1 = (x-1)(x-4)\left(\frac{A}{x-1} + \frac{B}{x-4}\right)$$

$1 = Ax - 4A + Bx - B$

$1 = -4A - B \qquad 0 = A + B$

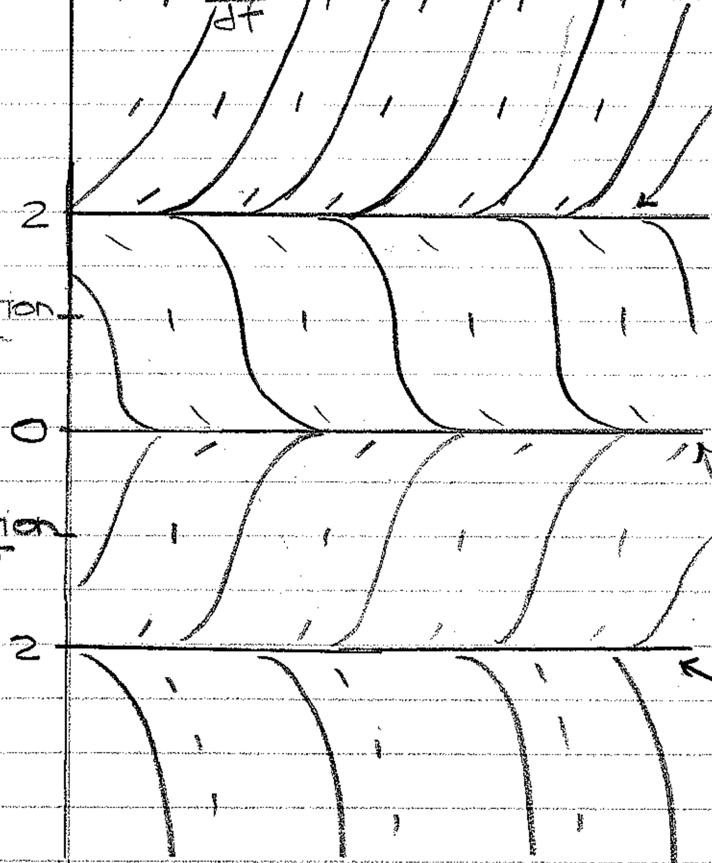
$B = -4A - 1 \qquad 1 = A - 4A$

$B = 1/3 \qquad A = -1/3$

$$\int\left(\frac{1}{x-4} - \frac{1}{x-1}\right)dx = \int 3\,dt \rightarrow \ln\left|\frac{x-4}{x-1}\right| = 3t + C$$

$$\frac{x-4}{x-1} = Ce^{3t} \qquad \boxed{x = \frac{4 - Ce^{3t}}{1 - Ce^{3t}}}$$

14. $\frac{dx}{dt} = x(x^2-4)$, critical points: $0, \pm 2$



$x=2$, Unstable Critical Point

$f(x) = dx/dt$

$f(4) = +48$

$f(3) = +15$

$f(1.5) = -2.625$

$f(0.5) = -1.875$

$f(-0.5) = 1.875$

$f(-1.5) = 2.625$

$f(-3) = -15$

$f(-4) = -48$

$x=0$, Stable Critical Point

$x=-2$, unstable critical Point

21. $dx/dt = kx - x^3 = x(k - x^2)$

A. if $k \le 0$ then the term $(k - x^2) \ne 0$ for all x
the only critical point when $k \le 0$ is when $x = 0$

$0 < a << 1$ $\quad$ $-1 << b < 0$

$dx/dt(a) < 0$ $\quad$ $dx/dt(b) > 0$ $\quad$ the point is STABLE

B. If $k > 0$, critical points exist @ $x = 0, \pm\sqrt{k}$

$x = 0$ $\quad$ $x = \pm\sqrt{k}$

$0 < a << 1$ $\quad$ $x^- \to \sqrt{k}$ $\quad$ $x^+ \to \sqrt{k}$

$dx/dt(a) > 0$ $\quad$ $dx/dt(\sqrt{k}) < 0$ $\quad$ $dx/dt(\sqrt{k}) > 0$

$-1 << b < 0$ $\quad$ $x^- \to -\sqrt{k}$ $\quad$ $x^+ \to \sqrt{k}$

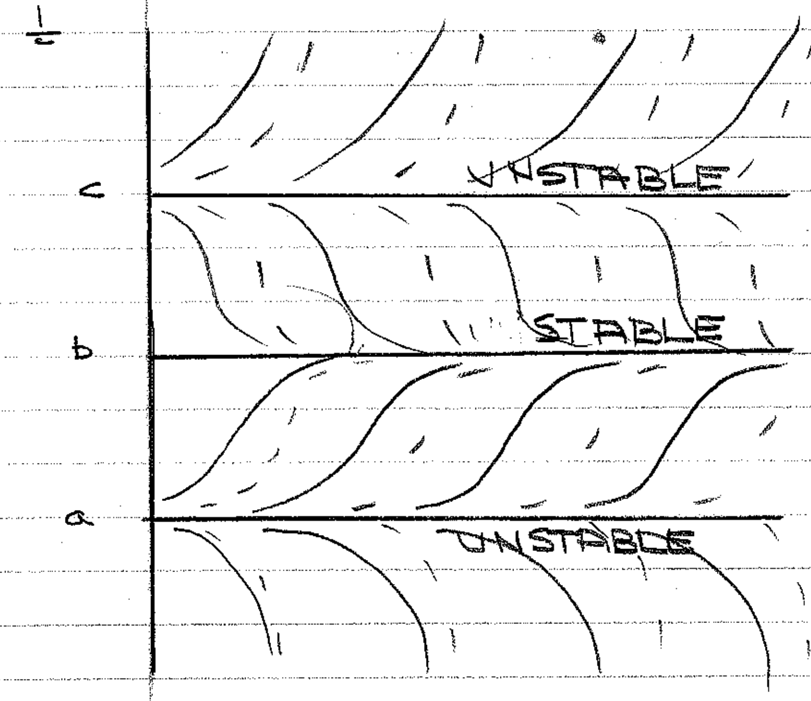
$dx/dt(b) < 0$ $\quad$ $dx/dt(-\sqrt{k}) < 0$ $\quad$ $dx/dt(-\sqrt{k}) > 0$ $\quad$ Stable Critical Points

unstable critical point

29. 1 $dx/dt = (x-a)(x-b)(x-c)$ Assume: $a < b < c$

2 $dx/dt = (a-x)(b-x)(c-x)$ (and that $a > 0$ for point eval.)

1



$x > c$ +

$b << x < c$ −

$b < x << c$ −

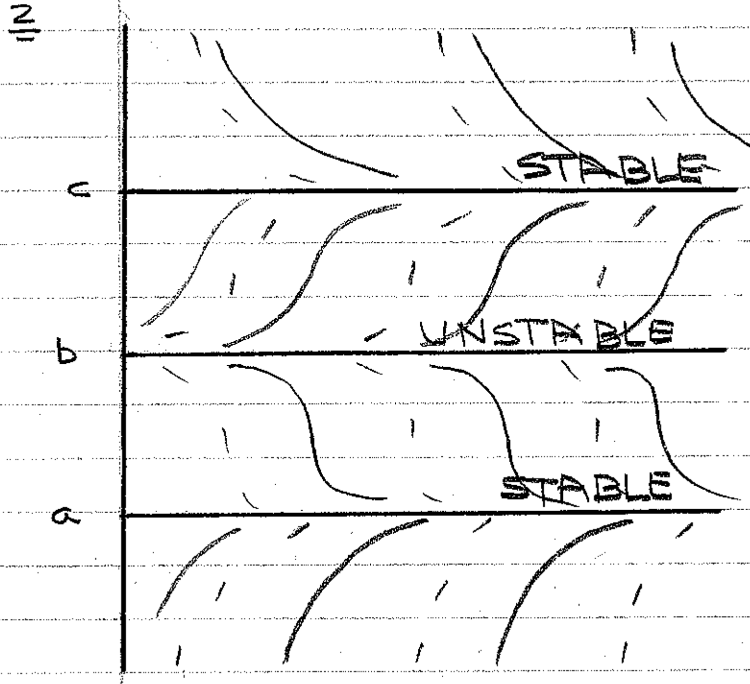
$a << x < b$ +

$a < x << b$ +

$a > x$ −

2 Funnels 1 Spout

2



2 Spouts 1 funnel

$x > c$ −

$b << x < c$ +

$b < x << c$ +

$a << x < b$ −

$a < x << b$ −

$a > x$ +

2.4: 4, 7

4. $y' = x - y \quad y(0) = 1 \quad y(x) = 2e^{-x} + x - 1$

$[0, \frac{1}{2}]$ $h = 0.25$

x_i	y_i	$f(x_i y_i)$	$y_i + h f(x_i y_i)$
0	1	-1	$1 + \frac{1}{4}(-1) = \frac{3}{4}$
$\frac{1}{4}$	$\frac{3}{4}$	$-\frac{1}{2}$	$\frac{3}{4} + \frac{1}{4}(-\frac{1}{2}) = \frac{5}{8}$
$\frac{1}{2}$	$\frac{5}{8}$	$-\frac{1}{8}$	$\frac{5}{8} + \frac{1}{4}(-\frac{1}{8}) = \frac{19}{32}$ $(= .594)$

$h = 0.1$

x_i	y_i	$f(x_i y_i)$	$y_i + h f(x_i y_i)$
0	1	-1	$1 + \frac{1}{10}(-1) = \frac{9}{10}$
$\frac{1}{10}$	$\frac{9}{10}$	$-\frac{4}{5}$	$\frac{9}{10} + \frac{1}{10}(-\frac{4}{5}) = \frac{41}{50}$
$\frac{2}{10}$	$\frac{41}{50}$	$-\frac{31}{50}$	$\frac{41}{50} + \frac{1}{10}(-\frac{31}{50}) = \frac{379}{500}$
$\frac{3}{10}$	$\frac{379}{500}$	$-\frac{79}{500}$	$\frac{379}{500} + \frac{1}{10}(-\frac{79}{500}) = \frac{3711}{5000}$
$\frac{4}{10}$	$\frac{3711}{5000}$	$-\frac{1711}{5000}$	$\frac{3711}{5000} + \frac{1}{10}(-\frac{1711}{5000}) = \frac{35399}{50000}$
$\frac{1}{2}$	$\frac{35399}{50000}$	$-\frac{10399}{50000}$	$\frac{35399}{50000} + \frac{1}{10}(-\frac{10399}{50000}) = .687182$

$y(\frac{1}{2}) = .71306$ The 2nd Approximation is much closer

7. $y' = -3x^2 y \quad y(0) = 3 \quad y = 3e^{-x^3}$

$[0, \frac{1}{2}]$ $h = \frac{1}{4}$

x_i	y_i	$f(x_i, y_i)$	$y_i + h f(x_i y_i)$
0	3	0	$3 + \frac{1}{4}(0) = 3$
$\frac{1}{4}$	3	$-\frac{9}{8}$	$3 + \frac{1}{4}(-\frac{9}{8}) = 2.719$
$\frac{1}{2}$	2.719	-2.039	$= 2.209$

$h = 0.1$

x_i	y_i	$f(x_i y_i)$	$y_i + h f(x_i y_i)$
0	3	0	3
.1	3	$-.09$	2.991
.2	2.991	$-.35892$	2.955
.3	2.955	$-.79788$	2.875
.4	2.875	-1.38802	2.737
.5	2.737	-2.05229	2.532

$y(\frac{1}{2}) = 2.647$

3.1: 4, 14, 23

4 $5x - 6y = 1 \longrightarrow x = \frac{1}{5}(1+6y)$

$6x - 5y = 10 \qquad \frac{6}{5}(1+6y) - 5y = 10$

$\frac{6}{5} + \frac{36}{5}y - \frac{25}{5}y = 10 \quad \frac{11}{5}y = \frac{44}{5} \quad y = 4, x = 5$

The system is consistent

14. (I) $2x + 7y + 3z = 19$ $\qquad$ $(2\text{I}) = 4x + 14y + 6z = 38$

(II) $4x + 9y + 12z = -1$ $\qquad$ $\text{II}'\ (\text{II} - 2\text{I}) = 0 - 5y + 6z = -39$

(III) $3x + y + 16z = -46$

$(\frac{3}{2}\text{I}) = 3x + \frac{21}{2}y + \frac{9}{2}z = \frac{57}{2}$

$\text{III}'\ (\text{III} - \frac{3}{2}\text{I}) = 0 - 9\frac{1}{2}y + 11\frac{1}{2}z = -74.5$

new system:

(I) $2x + 7y + 3z = 19$

(II') $-5y + 6z = -39$ $\qquad$ $(\frac{19}{20}\text{II}) = -9.5y + 11.4z = -74.1$

(III') $-9.5y + 11.5z = -74.5$ $\qquad$ $\text{III}''\ (\text{III}' - \frac{19}{20}\text{II}) = 0y + .1z = -.4$

new system: $\qquad$ soln:

(I) $2x + 7y + 3z = 19$ $\qquad$ $z = -4$

(II') $-5y + 6z = -39$ $\qquad$ $y = 3$

(III'') $+.1z = -.4$ $\qquad$ $x = 5$

The system is consistent

23. $y'' + 4y = 0 \quad y(x) = A\cos 2x + B\sin 2x$

$y(0) = 3 \quad y'(0) = 8$

$y' = 2B\cos 2x - 2A\sin 2x$

$y'' = -4B\sin 2x - 4A\cos 2x \quad = -4(A\cos 2x + B\sin 2x)$

$-4(A\cos x + B\sin x) + 4(A\cos x + B\sin x) = 0$

will $= 0$ for all A, B

$y'(0) = 8 = 2B\cos(0) - 2A\sin(0) = 2B$

$8 = 2B, \quad B = 4$

$y(0) = 3 = A\cos(0) + B\sin(0) = A$

$3 = A$

$y(x) = 3\cos 2x + \cancel{8}\ \sin 2x$

-DS

this soln. is correct

check: $-4(3\cos 2x + 8\sin 2x) + 4(3\cos 2x + 8\sin 2x) = 0$