Ashley Birgit Dalgaard 324

HW: 3.6 # 3, 10, 14, 25, 28, 36, 38, 52

4.1 # 11, 16, 20, 22, 28, 32, 35

3) $\begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 0 & 5 & 0 \\ 3 & 6 & 9 & 8 \\ 4 & 0 & 10 & 7 \end{vmatrix} = 1 \begin{vmatrix} 0 & 5 & 0 \\ 6 & 9 & 8 \\ 0 & 10 & 7 \end{vmatrix} + 0 + 0 + 0 = 1[0 - 5(42-0) + 0]$

$= 1(-5 \times 42) = \boxed{-210}$

14/15

10) $\begin{vmatrix} -3 & 6 & 5 \\ 1 & -2 & -4 \\ 2 & -5 & 12 \end{vmatrix} \xrightarrow{c_2+2c_1} \begin{vmatrix} -3 & 0 & 5 \\ 1 & 0 & -4 \\ 2 & -1 & 12 \end{vmatrix} = 0 + 0 + 1(12-5) = \boxed{7}$

check: $-3(-24-20) - 6(12+8) + 5(-5+4)$

$= -3(-44) - 6(20) + 5(-1)$

$= 132 - 120 - 5 = 7$ ✓

14) $\begin{vmatrix} 4 & 2 & -2 \\ 3 & 1 & -5 \\ -5 & -4 & 3 \end{vmatrix} \rightarrow (2) \begin{vmatrix} 2 & 1 & -1 \\ 3 & 1 & -5 \\ -5 & -4 & 3 \end{vmatrix}$

det $= 2[2(3-20) - 1(9-25) - 1(-12+5)] = 2[2(-17) - 1(-16) - 1(-7)] =$

$2(-34+16+7) = 2(-11) = \boxed{-22}$

-34
$+16$
-18
$+7$
-11

check: $4(3-20) - 2(9-25) - 2(-12+5) = 4(-17) - 2(-16) - 2(-7) =$

$-68 + 32 + 14 = -22$ ✓

cramer

25) $5x+6y=12$
$3x+4y=6$

$\det = 20-18 = 2 \quad A = \begin{pmatrix} 5 & 6 \\ 3 & 4 \end{pmatrix}$

$x = \dfrac{\begin{vmatrix} 12 & 6 \\ 6 & 4 \end{vmatrix}}{2} = \dfrac{48-36}{2} = \dfrac{12}{2} = \boxed{6 = x}$

$y = \dfrac{\begin{vmatrix} 5 & 12 \\ 3 & 6 \end{vmatrix}}{2} = \dfrac{30-36}{2} = \dfrac{-6}{2} = \boxed{-3 = y}$

28) $5x_1 + 4x_2 - 2x_3 = 4$
$2x_1 + 3x_3 = 2$
$2x_1 - x_2 - x_3 = 1$

$A = \begin{pmatrix} 5 & 4 & -2 \\ 2 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix} \quad |A| = 5(3) - 4(2-6) - 2(-2) = 15 + 16 + 4 = 35$

$x_1 = \frac{1}{35}\begin{vmatrix} 4 & 4 & -2 \\ 2 & 0 & 3 \\ 1 & -1 & 1 \end{vmatrix} = \frac{1}{35}\left(4(3) - 4(2-3) - 2(-2)\right) = \frac{1}{35}(12+4+4) = \frac{20}{35} = \boxed{\frac{4}{7} = x_1}$

$x_2 = \frac{1}{35}\begin{vmatrix} 5 & 4 & -2 \\ 2 & 2 & 3 \\ 2 & 1 & 1 \end{vmatrix} = \frac{1}{35}\left(5(2-3) - 4(2-6) - 2(2-4)\right) = \frac{1}{35}(-5+16+4) = \frac{15}{35} = \boxed{\frac{3}{7} = x_2}$

$x_3 = \frac{1}{35}\begin{vmatrix} 5 & 4 & 4 \\ 2 & 0 & 2 \\ 2 & -1 & 1 \end{vmatrix} = \frac{1}{35}\left(5(2) - 4(2-4) + 4(-2)\right) = \frac{1}{35}(10+8-8) = \frac{10}{35} = \boxed{\frac{2}{7} = x_3}$

36) $A = \begin{pmatrix} -4 & 4 & 3 \\ 3 & -1 & -5 \\ -1 & 0 & -5 \end{pmatrix} \xrightarrow{R_2 - R_3} \begin{pmatrix} -4 & 4 & 3 \\ 2 & -1 & 0 \\ 1 & 0 & -5 \end{pmatrix} \xrightarrow{R_1 + 4R_2} \begin{pmatrix} 0 & 2 & 3 \\ 2 & -1 & 0 \\ 1 & 0 & -5 \end{pmatrix} \quad |A| = 2(-10) + 3(1) = 20 + 3 = \boxed{23}$

$A_{ij} = \begin{pmatrix} (5) & -(-15+5) & (0+1) \\ -(-20) & (20-3) & -(0-4) \\ (20+3) & -(20-9) & (4-12) \end{pmatrix} = \begin{pmatrix} 5 & 10 & 1 \\ 20 & 17 & 4 \\ -17 & -11 & -8 \end{pmatrix}$

$A_{ij}^T = \begin{pmatrix} 5 & 20 & -17 \\ 10 & 17 & -11 \\ 1 & 4 & -8 \end{pmatrix} \quad \boxed{A^{-1} = \dfrac{\begin{pmatrix} 5 & 20 & -17 \\ 10 & 17 & -11 \\ 1 & 4 & -8 \end{pmatrix}}{23}}$

38) $A = \begin{pmatrix} 3 & 4 & -3 \\ 3 & 2 & -1 \\ -3 & 2 & -4 \end{pmatrix} \xrightarrow[R_2 + R_3]{R_1 + R_3} \begin{pmatrix} 0 & 6 & -7 \\ 0 & 4 & -5 \\ -3 & 2 & -4 \end{pmatrix} \quad |A| = +3(-30+28) = -6$

$A_{ij} = \begin{pmatrix} -6 & 15 & 12 \\ 10 & -21 & -18 \\ 2 & -6 & -6 \end{pmatrix} \quad A_{ij}^T = \begin{pmatrix} -6 & 10 & 2 \\ 15 & -21 & -6 \\ 12 & -18 & -6 \end{pmatrix} \quad \boxed{A^{-1} = \dfrac{\begin{pmatrix} -6 & 10 & 2 \\ 15 & -21 & -6 \\ 12 & -18 & -6 \end{pmatrix}}{6}}$

52) $A^T = A^{-1}$ orthogonal

$|A^T| = |A|$ Good.

$|A^{-1}| = |A|^{-1} = \frac{1}{|A|}$

$A^T = A^{-1} \rightarrow |A^T| = |A^{-1}|$

So $|A| = 1/|A|$

$|A|^2 = 1$

$|A| = \pm 1$

4.11) $u = (5,7)$ $v = (2,3)$ $w = (1,1)$

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}5\\7\end{pmatrix}s + \begin{pmatrix}2\\3\end{pmatrix}t$$

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{bmatrix}5&2\\7&3\end{bmatrix}\begin{pmatrix}s\\t\end{pmatrix}$$ inverse = $\begin{pmatrix}3&-2\\-7&5\end{pmatrix}$

det = 15-14 = 1 ✓

$$\begin{pmatrix}s\\t\end{pmatrix} = \begin{pmatrix}3&-2\\-7&5\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\end{pmatrix}$$ $s = 1$, $t = -2$

$\boxed{w = u - 2v}$

16) $u = (5,-2,4)$ $v = (2,-3,5)$ $w = (4,5,-7)$

$$a\begin{pmatrix}5\\-2\\4\end{pmatrix} + b\begin{pmatrix}2\\-3\\5\end{pmatrix} + c\begin{pmatrix}4\\5\\-7\end{pmatrix} = \vec{0}$$ $$\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}5&2&4\\-2&-3&5\\4&5&-7\end{pmatrix}^{-1}\begin{pmatrix}0\\0\\0\end{pmatrix}$$ $$\begin{pmatrix}a\\b\\c\end{pmatrix} = \vec{0}$$

$$\begin{pmatrix}5a\\-2a\\4a\end{pmatrix} + \begin{pmatrix}2b\\-3b\\5b\end{pmatrix} + \begin{pmatrix}4c\\5c\\-7c\end{pmatrix} = \vec{0}$$

det $= 5(21-25) - 2(14-20) + 4(-10+12)$

$$\begin{pmatrix}5&2&4\\-2&-3&5\\4&5&-7\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$ $= 5(-4) - 2(-6) + 4(2) = -20+12+8 = 0$

linearly dependent

20) $u = (5,5,4)$ $v = (2,3,1)$ $w = (4,1,5)$

$$\begin{pmatrix}5&2&4\\5&3&1\\4&1&5\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$ linearly dependent

GE: $R_2 - R_1$ $$\begin{pmatrix}5&2&4\\0&1&-3\\4&1&5\end{pmatrix} \xrightarrow{R_3 - R_2} \begin{pmatrix}5&2&4\\0&1&-3\\4&0&8\end{pmatrix} \xrightarrow{C_1 - \frac{1}{2}C_3} \begin{pmatrix}3&2&4\\3/2&1&-3\\0&0&8\end{pmatrix} \xrightarrow{R_2 - \frac{1}{2}R_1} \begin{pmatrix}3&2&4\\0&0&-5\\0&0&8\end{pmatrix} \xrightarrow[-\frac{1}{5}R_2]{\frac{1}{8}R_3} \begin{pmatrix}3&2&4\\0&0&1\\0&0&1\end{pmatrix}$$

$$\xrightarrow{R_3 - R_2} \begin{pmatrix}3&2&4\\0&0&1\\0&0&0\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$ $c = 1$ $a = -2$

$b = 3$

22) $u=(1,1,0)$ $v=(5,1,3)$ $w=(0,1,2)$

$$\begin{pmatrix}1&5&0\\1&1&1\\0&3&2\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix}$$

GE: $\begin{pmatrix}1&5&0\\1&1&1\\0&3&2\end{pmatrix}\xrightarrow{R_2-R_1}\begin{pmatrix}1&5&0\\0&-4&1\\0&3&2\end{pmatrix}\rightarrow$

$a+5b=0$

$-4b+c=0 \rightarrow 4b=c \rightarrow b=\frac{1}{4}c$

$3b+2c=0 \qquad 3b=-2c \rightarrow b=-\frac{2}{3}c \quad \neq$

28) $t=(7,7,7)$ $u=(2,5,3)$ $v=(4,1,-1)$ $w=(1,1,5)$

$$\begin{pmatrix}2&4&1\\5&1&1\\3&-1&5\end{pmatrix}\begin{pmatrix}a\\b\\c\end{pmatrix}=\begin{pmatrix}7\\7\\7\end{pmatrix}$$

GE: $\begin{pmatrix}2&4&1\\5&1&1\\3&-1&5\end{pmatrix}\xrightarrow[R_2+R_3]{R_1-R_2}\begin{pmatrix}-3&3&0\\5&1&1\\8&0&6\end{pmatrix}\xrightarrow[\frac{1}{2}R_3]{\frac{1}{3}R_1}\begin{pmatrix}-1&1&0\\5&1&1\\4&0&3\end{pmatrix}\xrightarrow{R_3+4R_1}\begin{pmatrix}-1&1&0\\5&1&1\\0&4&3\end{pmatrix}\xrightarrow{R_2-R_1}\begin{pmatrix}-1&1&0\\6&0&1\\0&4&3\end{pmatrix}$

$-a+b=7 \rightarrow b=7+a$

$6a+c=7 \rightarrow c=7-6a$

$4b+3c=7 \longrightarrow 4(7+a)+3(7-6a)=7$

$28+4a+21-18a=7$

$49-14a=7$

$14a=42$ $\boxed{a=3}$

$\boxed{b=10}$

$c=7-18=\boxed{-11=c}$

32) V set all (x,y,z): $z=2x+3y$

✓ addition: $\begin{pmatrix}u_1\\u_2\\2u_1+3u_2\end{pmatrix}+\begin{pmatrix}v_1\\v_2\\2v_1+3v_2\end{pmatrix}=\begin{pmatrix}u_1+v_1\\u_2+v_2\\(2u_1+2v_1)+(3u_2+3v_2)\end{pmatrix}=\begin{pmatrix}u_1+v_1\\u_2+v_2\\2(u_1+v_1)+3(u_2+v_2)\end{pmatrix}$

✓ mult: $c\begin{pmatrix}u_1\\u_2\\2u_1+3u_2\end{pmatrix}=\begin{pmatrix}cu_1\\cu_2\\2cu_1+3cu_2\end{pmatrix}$

33) V set all (x,y,z) : $z\geq 0$

ex. $\begin{pmatrix}1\\1\\0\end{pmatrix}+-\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}0\\0\\-1\end{pmatrix}$

mult: $-1\begin{pmatrix}1\\1\\1\end{pmatrix}=\begin{pmatrix}-1\\-1\\-1\end{pmatrix}$