Helena Menendez
Math 320
10/20/10

Homework # : Wednesday

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§4.2

6) $\mathbb{R}^4$; $x_1 = 3x_3$; $x_2 = 4x_4$ (x_3 & x_4 = free)

a) $\overset{\vec{u}}{\begin{bmatrix} 3x_3 \\ 4x_4 \\ x_3 \\ x_4 \end{bmatrix}} + \overset{\vec{v}=2\vec{u}}{\begin{bmatrix} 6x_3 \\ 8x_4 \\ 2x_3 \\ 2x_4 \end{bmatrix}} = \overset{\vec{u}+\vec{v}}{\begin{bmatrix} 9x_3 \\ 12x_4 \\ 3x_3 \\ 3x_4 \end{bmatrix}} = 3\begin{bmatrix} 3x_3 \\ 4x_4 \\ x_3 \\ x_4 \end{bmatrix}$ closed under addition ✓

b) $-1\begin{bmatrix} 3x_3 \\ 4x_4 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -3x_3 \\ -4x_4 \\ -x_3 \\ -x_4 \end{bmatrix}$ closed under mult. ✓

$\boxed{\text{yes, } W \text{ is a subspace of } \mathbb{R}^4}$ ✓

11) $x_1 + x_2 = x_3 + x_4$ $\quad x_4 = x_1 + x_2 - x_3$

a) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_1+x_2-x_3 \end{bmatrix} + \begin{bmatrix} 3x_1 \\ 3x_2 \\ 3x_3 \\ 3x_1+3x_2-3x_3 \end{bmatrix} = \begin{bmatrix} 4x_1 \\ 4x_2 \\ 4x_3 \\ 4x_1+4x_2-4x_3 \end{bmatrix} = 4\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_1+x_2-x_3 \end{bmatrix}$ closed under add ✓

b) $3\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_1+x_2-x_3 \end{bmatrix}$ closed under mult. ✓

$\boxed{\text{yes, } W \text{ is a subspace of } \mathbb{R}^4}$ ✓

12) $x_1x_2 = x_3x_4$

$\vec{x} = \vec{x}_1\vec{x}_2 = \vec{x}_3\vec{x}_4$

a) $\vec{y} = \vec{y}_1\vec{y}_2 = \vec{y}_3\vec{y}_4$

$\vec{x}+\vec{y}$: $(\vec{x}_1+\vec{y}_1)(\vec{x}_2+\vec{y}_2) = (\vec{x}_3+\vec{y}_3)(\vec{x}_4+\vec{y}_4)$

$\vec{x}_1\vec{x}_2 + \vec{x}_1\vec{y}_2 + \vec{y}_1\vec{x}_2 + \vec{y}_1\vec{y}_2 = \vec{x}_3\vec{x}_4 + \vec{x}_3\vec{y}_4 + \vec{y}_3\vec{x}_4 + \vec{y}_3\vec{y}_4$

$\boxed{\text{No! Not closed under addition} \rightarrow W \text{ is NOT a subspace of } \mathbb{R}^4}$ ✓

16) $\left[\begin{array}{cccc|c} 1 & -4 & -3 & -7 & 0 \\ 2 & -1 & 1 & 7 & 0 \\ 1 & 2 & 3 & 11 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -3 & -7 & 0 \\ 0 & 7 & 7 & 21 & 0 \\ 0 & 6 & 6 & 18 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -3 & -7 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 1 & 1 & 3 & 0 \end{array}\right]$

$\Rightarrow \left[\begin{array}{cccc|c} 1 & -4 & -3 & -7 & 0 \\ 0 & 1 & 1 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$ so

$c_1 = \overset{-4s-12t}{4(-s-3t)} + 3s + 7t = -s - 5t$

$c_2 = -s - 3t$

$c_3 = s$

$c_4 = t$

completed on reverse

1c continued) $c_1 = -s - 5t$
$c_2 = -s - 3t$
$c_3 = s$
$c_4 = t$

$$\vec{x} = \begin{bmatrix} -s-5t \\ -s-3t \\ s \\ t \end{bmatrix} = s \overset{\vec{u}}{\begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}} + t \overset{\vec{v}}{\begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}}$$

$$\boxed{\text{so: } \vec{u} = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} \ \& \ \vec{v} = \begin{bmatrix} -5 \\ -3 \\ 0 \\ 1 \end{bmatrix}}$$

2c)
$$\left[\begin{array}{cccc|c} 1 & 5 & 1 & -8 & 0 \\ 2 & 5 & 0 & -5 & 0 \\ 2 & 7 & 1 & -9 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 5 & 1 & -8 & 0 \\ 0 & -5 & -2 & 11 & 0 \\ 0 & -3 & -1 & 7 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 0 \\ 0 & -5 & -2 & 11 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{7}{3} & 0 \end{array}\right]$$

$$\Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -1 & 3 & 0 \\ 0 & 1 & \frac{1}{3} & -\frac{7}{3} & 0 \\ 0 & 0 & -\frac{1}{3} & -\frac{2}{3} & 0 \end{array}\right] \Rightarrow \left[\begin{array}{cccc|c} 1 & 0 & 0 & 5 & 0 \\ 0 & 1 & 0 & -3 & 0 \\ 0 & 0 & +1 & +2 & 0 \end{array}\right]$$

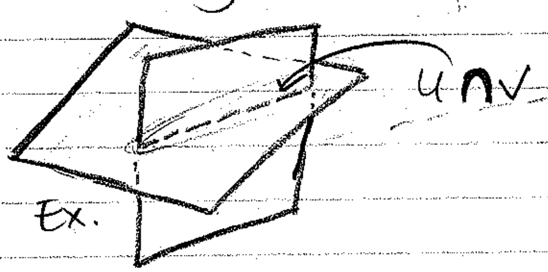
$c_1 = -5t$
$c_2 = 3t$
$c_3 = -2t$
$c_4 = t$

so: $t\vec{u} = t\begin{bmatrix} -5 \\ 3 \\ -2 \\ 1 \end{bmatrix}$; $\boxed{\vec{u} = \begin{bmatrix} -5 \\ 3 \\ -2 \\ 1 \end{bmatrix}}$

3a) Given: U & V are subspaces of W so they are closed under add. & mult.
Show: $U \cap V$ is a subspace of W and " " mult.

a) Let $\vec{x}$ & $\vec{y} \in U$ and $\vec{x}$ & $\vec{y} \in V$
so: $\vec{x}+\vec{y} \in U$ and $\vec{x}+\vec{y} \in V$
If: $\vec{x}+\vec{y} \in U$ and $\in V$, then
$U \cap V$ $\vec{x}+\vec{y} \in U \cap V$

b) Let $\vec{w} \in U \cap V$, so $\vec{w} \in U$ & $\vec{w} \in V$
Let $c \in \mathbb{R}$
$c\vec{w} \in U \cap V$ b/c closed under mult.
so: $c\vec{w} \in U$ and $c\vec{w} \in V$
because they are also closed under mult



Ex.

§ 4.3

12) $\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 3 & -2 & -4 \\ -1 & -1 & -3 \\ 9 & -3 & 3 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & -1 & -3 \\ 0 & 5 & 25 \\ 0 & 1 & 5 \\ 0 & 6 & 30 \end{array}\right] \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

so: $\boxed{\vec{w} = 2\vec{v}_1 + 5\vec{v}_2}$

14) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -3 \\ 0 & -2 & 1 & 2 \\ 3 & 0 & 1 & -3 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -6 \\ 0 & -2 & 0 & 11 \\ 0 & 0 & 1 & -9 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & -9 \end{array}\right]$ undefined!

$\vec{w}$ $\boxed{\text{can't}}$ be a lin. comb. of $\vec{v}_1, \vec{v}_2, \text{ \& } \vec{v}_3$

16) $\left[\begin{array}{ccc|c} 2 & 3 & 1 & 4 \\ -1 & 0 & 2 & 5 \\ 4 & 1 & -1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 3 & 5 & 14 \\ 0 & 1 & 7 & 26 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -2 & -5 \\ 0 & 1 & 7 & 26 \\ 0 & 0 & -16 & -64 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 4 \end{array}\right]$

so: $\boxed{\vec{w} = 3\vec{v}_1 - 2\vec{v}_2 + 4\vec{v}_3}$

18) $\left[\begin{array}{ccc|c} \cancel{2}\ 1 & \cancel{4}\ 2 & \cancel{-2}\ -1 & 0 \\ 0 & -5 & 1 & 0 \\ -3 & -6 & 3 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -1 & 0 \\ 0 & 1 & -1/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -3/5 & 0 \\ 0 & 1 & -1/5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$c_1 = \frac{3}{5}t$ so: let $t = 5$: $\boxed{3\vec{v}_1 + \vec{v}_2 + 5\vec{v}_3 = \vec{0}}$

$c_2 = \frac{1}{5}t$

$c_3 = t$

20) $\left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ -1 & 1 & 1 & 0 \\ -1 & 1 & 4 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 3 & 7 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 2 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

yes, $\vec{v}_1, \vec{v}_2, \text{ \& } \vec{v}_3$ are $\boxed{\text{lin. indep}}$ because only the trivial solution yields the zero vector!