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HW #9

13/13

4.4 2, 8, 14, 20, 24

#2 NOT INDEPENDENT B/C V_1 IS A SCALAR MULTIPLE OF V_2
$\rightarrow$ NO BASIS.

#8
$$\begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&7&4\\0&0&6&5\end{bmatrix} \xrightarrow{R_3-R_4} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&-1\\0&0&6&5\end{bmatrix} \xrightarrow{R_4-6R_3} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&-1\\0&0&0&11\end{bmatrix} \xrightarrow{11R_3+R_4} \begin{bmatrix}1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{bmatrix}$$

$\therefore$ CAN BE INDEPENDENT SO... BASIS FOR $\mathbb{R}^4$

#14 $a=-2b$; $c=-3d$ $\quad V=(-2b, b, -3d, d)=b(-2,1,0,0)+d(0,0,-3,1)$

$\therefore$ 2-DIMENSIONAL W/ BASIS CONSISTING OF $V_1=(-2,1,0,0)$ & $V_2=(0,0,-3,1)$

#20
$$\begin{bmatrix}1&-3&-10&5\\1&4&11&-2\\1&3&8&-1\end{bmatrix} \xrightarrow{R_2-R_1} \begin{bmatrix}1&-3&-10&5\\0&7&21&-7\\1&3&8&-1\end{bmatrix} \xrightarrow{R_3-R_1} \begin{bmatrix}1&-3&-10&5\\0&1&3&-1\\0&6&18&-6\end{bmatrix} \xrightarrow{R_3-6R_2} \begin{bmatrix}1&-3&-10&5\\0&1&3&-1\\0&0&0&0\end{bmatrix} \xrightarrow{R_1+3R_2}$$

$$\begin{bmatrix}1&0&-4&2\\0&1&3&-1\\0&0&0&0\end{bmatrix}$$
$x_3=s$; $x_4=t$; $x_1=4s-2t$; $x_2=-3s+t$

$x=s(4,-3,1,0)+t(-2,1,0,1)$

$\therefore$ 2 DIMENSIONAL W/ BASIS $V_1=4,-3,1,0$ $\quad V_2=-2,1,0,1$

#24
$$\begin{bmatrix}1&3&-4&-8&6\\1&0&2&1&3\\2&7&-10&-19&13\end{bmatrix} \xrightarrow[R_3-2R_1]{R_2-R_1} \begin{bmatrix}1&3&-4&-8&6\\0&-3&6&9&-3\\0&1&-2&-3&1\end{bmatrix} \xrightarrow{R_2+3R_3} \begin{bmatrix}1&3&-4&-8&6\\0&0&0&0&0\\0&1&-2&-3&1\end{bmatrix} \xrightarrow[R_1-3R_3]{\text{SWITCH } R_2 \text{ \& } R_3}$$

$$\begin{bmatrix}1&0&2&1&3\\0&1&-2&-3&1\\0&0&0&0&0\end{bmatrix}$$
x_3 ; x_4 & x_5 FREE VARIABLES

$x_3=s$; $x_4=t$; $x_5=r$; $x_1=-2s+-t+-3r$

$x_2=2s+3t+r$

$\therefore$ 3-DIMENSIONAL BASIS W/ $V_1=(-2,2,1,0,0)$

$V_2=(-1,3,0,1,0)$; $V_3=(-3,1,0,0,1)$

$\boxed{4.5}$ 2, 8, 13, 15

#2

$$\begin{bmatrix}5&2&4\\2&1&1\\4&1&5\end{bmatrix}\xrightarrow{R_3-2R_2}\begin{bmatrix}5&2&4\\2&1&1\\0&-1&3\end{bmatrix}\xrightarrow{R_1-2.5R_2}\begin{bmatrix}0&.5&1.5\\2&1&1\\0&-1&3\end{bmatrix}\xrightarrow[R_1-R_2]{\text{SWITCH } R_1 \,\&\, R_2}\begin{bmatrix}1&0&-1\\0&1&3\\0&-1&3\end{bmatrix}\xrightarrow{R_3+R_2}$$

$$\begin{bmatrix}1&0&-1\\0&1&3\\0&0&0\end{bmatrix}$$

ROW BASIS: 1^{ST} & 2^{ND} VECTORS FOR E

COLUMN BASIS: 1^{ST} & 2^{ND} VECTORS FOR A

#8

$$\begin{bmatrix}1&-2&-3&-5\\1&4&9&2\\1&3&7&1\\2&2&6&-3\end{bmatrix}\xrightarrow[R_4-2R_1]{R_2-R_1,\ R_3-R_1}\begin{bmatrix}1&-2&-3&-5\\0&6&12&7\\0&5&10&6\\0&6&12&7\end{bmatrix}\xrightarrow[R_2-R_3]{R_4-R_2}\begin{bmatrix}1&-2&-3&-5\\0&1&2&1\\0&5&10&6\\0&0&0&0\end{bmatrix}\xrightarrow[R_1+2R_2]{R_3-5R_2}$$

$$\begin{bmatrix}1&0&1&-3\\0&1&2&1\\0&0&0&1\\0&0&0&0\end{bmatrix}\xrightarrow[R_1+3R_3]{R_2-R_3}\begin{bmatrix}1&0&1&0\\0&1&2&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$$

ROW BASIS: 1^{ST}, 2^{ND} & 4^{TH} VECTORS FOR E.

COLUMN BASIS: 1^{ST}, 2^{ND} & 3^{RD} VECTORS.

#13

$$\begin{bmatrix}1&2&5\\3&-1&1\\-2&3&4\\4&2&8\end{bmatrix}\xrightarrow[R_4-4R_1]{R_2-3R_1,\ R_3+2R_1}\begin{bmatrix}1&2&5\\0&-7&-14\\0&7&14\\0&-6&-12\end{bmatrix}\xrightarrow{R_3+R_2}\begin{bmatrix}1&2&5\\0&-1&-2\\0&0&0\\0&-1&-2\end{bmatrix}\xrightarrow{R_4-R_2}\begin{bmatrix}1&2&5\\0&-1&-2\\0&0&0\\0&0&0\end{bmatrix}\xrightarrow{R_1+2R_2}$$

$$\begin{bmatrix}1&0&1\\0&1&2\\0&0&0\\0&0&0\end{bmatrix}$$

LINEARLY INDEPENDENT: V_1 & V_2

#15

$$\begin{bmatrix}3&2&4&1\\2&1&3&2\\2&2&2&3\\2&1&3&4\end{bmatrix}\xrightarrow[R_1-R_2,\ R_4-R_2]{R_3-R_2}\begin{bmatrix}1&1&1&-1\\2&1&3&2\\0&1&-1&1\\0&0&0&2\end{bmatrix}\xrightarrow{R_2-2R_1}\begin{bmatrix}1&1&1&-1\\0&-1&1&4\\0&-1&1&1\\0&0&0&2\end{bmatrix}\xrightarrow[R_3-R_2]{R_1+R_2}\begin{bmatrix}1&0&2&-3\\0&-1&1&4\\0&0&0&-3\\0&0&0&2\end{bmatrix}$$

$$\rightarrow\begin{bmatrix}1&0&2&-3\\0&-1&1&4\\0&0&0&-1\\0&0&0&1\end{bmatrix}\xrightarrow[R_2+4R_3,\ R_4+R_3]{R_1-3R_3}\begin{bmatrix}1&0&2&0\\0&-1&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}$$

LINEARLY INDEPENDENT: V_1, V_2 & V_4

$\boxed{4.6}$ 2, 15, 19, 23

#2 $V_1 \cdot V_2 = 3\cdot 6 + -2\cdot 3 + 4\cdot 3 + -4\cdot 6 = 18 + \cancel{-12} + \cancel{12} + -24 \neq 0$

∴ NO, THEY ARE NOT MUTUALLY ORTHOGONAL

#15 $V_1 = (1, -2, -3, 5)$ so... $x_1 - 2x_2 - 3x_3 + 5x_4 = 0$

$x_2 = r$; $x_3 = s$; $x_4 = t$; $x_1 = 2r + 3s - 5t$

POSSIBLE SOLUTIONS: $x_1 = 2, 3, -5$; $x_2 = 1, 0, 0$; $x_3 = 0, 1, 0$

$x_4 = 0, 0, 1$

$u_1 = (2, 1, 0, 0)$; $u_2 (3, 0, 1, 0)$; $u_3 = (-5, 0, 0, 1)$

#19 $V_1 = (1, 2, 5, 2, 3)$; $V_2 = (3, 7, 11, 9, 5)$

$$\begin{pmatrix} 1 & 2 & 5 & 2 & 3 \\ 3 & 7 & 11 & 9 & 5 \end{pmatrix} \xrightarrow{R_2 - 3R_1} \begin{matrix} 1 & 2 & 5 & 2 & 3 \\ 0 & 1 & -4 & 3 & -4 \end{matrix} \xrightarrow{R_1 - 2R_2} \begin{matrix} 1 & 0 & 13 & -4 & 11 \\ 0 & 1 & -4 & 3 & -4 \end{matrix}$$

$x_3 = r$; $x_4 = s$; $x_5 = t$; $x_2 = 4r - 3s + 4t$; $x_1 = -13r + 4s - 11t$

$u_1 = (-13, 4, 1, 0, 0)$; $u_2 = (4, -3, 0, 1, 0)$; $u_3 = (-11, 4, 0, 0, 1)$

#23 a. $|u+v|^2 + |u-v|^2 = 2|u|^2 + 2|v|^2 = (u^2 + \cancel{2uv} + v^2) + (u^2 - \cancel{2uv} + v^2)$

$= 2|u|^2 + 2|v|^2$

b. $|u+v|^2 - |u-v|^2 = (\cancel{u^2} + 2uv + \cancel{v^2}) - (\cancel{u^2} - 2uv + \cancel{v^2})$

$= 4uv$