

Random Variables

<u>Name (parameters)</u>	<u>probability function</u>	<u>expectation</u>	<u>variation</u>	<u>description</u>
<u>Discrete</u>				
Bernoulli (p)	$p(1) = p$ $p(0) = 1-p$	p	$p(1-p)$	A binary indicator of the success of an experiment successful w/ prob. p .
Binomial (n, p)	$p(i) = \binom{n}{i} p^i (1-p)^{n-i}$	np	$np(1-p)$	The number of successes that occur in n trials of an experiment with success probability p .
Poisson (λ)	$P\{X=i\} = e^{-\lambda} \frac{\lambda^i}{i!}$	λ	λ	The number of events (i) that will occur (randomly) in a time interval in which λ events would be expected to occur. Note: $\lambda = \frac{A}{t}$, where λ = average rate of occurrence.
Geometric (p)	$p(n) = (1-p)^{n-1} p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$	The number of trials (n) needed to get a success, where p is the probability that any trial succeeds,
Negative Binomial (r, p)	$p(n) = \binom{n-1}{r-1} p^r (1-p)^{n-r}$	$\frac{r}{p}$	$r \frac{(1-p)}{p^2}$	The number of trials (n) needed to get r successes where p is the probability of success in any trial
Hypergeometric (n, N, m)	$p(i) = \frac{\binom{m}{i} \binom{N-m}{n-i}}{\binom{N}{n}}$	np	$\frac{(N-n)}{N-1} np(1-p)$	The number (i) of members of a subpopulation of size m chosen when a sample of size n is chosen from a population of size N .
<u>Notes</u>				
① $i \leq N-m$				
② $i \leq m$ $\Leftrightarrow n-(N-m) \leq i \leq \min(n, m)$				
③ $i \leq n$				
<u>Continuous</u>				
Uniform (α, β)	$f(x) = \begin{cases} \frac{1}{\beta-\alpha} & \alpha < x < \beta \\ 0 & \text{otherwise} \end{cases}$	$\frac{\alpha+\beta}{2}$	$\frac{(\beta-\alpha)^2}{12}$	A random variable evenly distributed over an interval
Normal (μ, σ^2)	$f(x) = \frac{e^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\sqrt{2\pi\sigma^2}}$	μ	σ^2	The limiting case of the sum of a large number of largely independent random variables none of which dominates.
Exponential (λ)	$f(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq 0 \\ 0 & t < 0 \end{cases}$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$	The amount of time needed for an event to occur, given that events occur at rate λ .
$\lambda(t)$	$F(t) = 1 - e^{-\int_0^t \lambda(t) dt}$			
	$f(t) = \lambda(t) e^{-\int_0^t \lambda(u) du}$			
Gamma (r, λ)	$f(t) = \frac{\lambda^r e^{-\lambda t} (\lambda t)^{r-1}}{\Gamma(r)}$	$\frac{r}{\lambda}$	$\frac{r}{\lambda^2}$	The amount of time until r events occur, if they occur at rate λ .
	where $\Gamma(r) = \int_0^\infty e^{-x} x^{r-1} dx$			
	$= (r-1)!$			
Beta (a, b)	$f(x) = \begin{cases} \frac{1}{B(a,b)} x^{a-1} (1-x)^{b-1} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2 (a+b+1)}$	The conditional success probability given that $a+b$ trials result in a successes (b fails).
	where $B(a,b) = \int_0^1 x^{a-1} (1-x)^{b-1} dx$			
	$= \frac{T(a) T(b)}{T(a+b)}$			