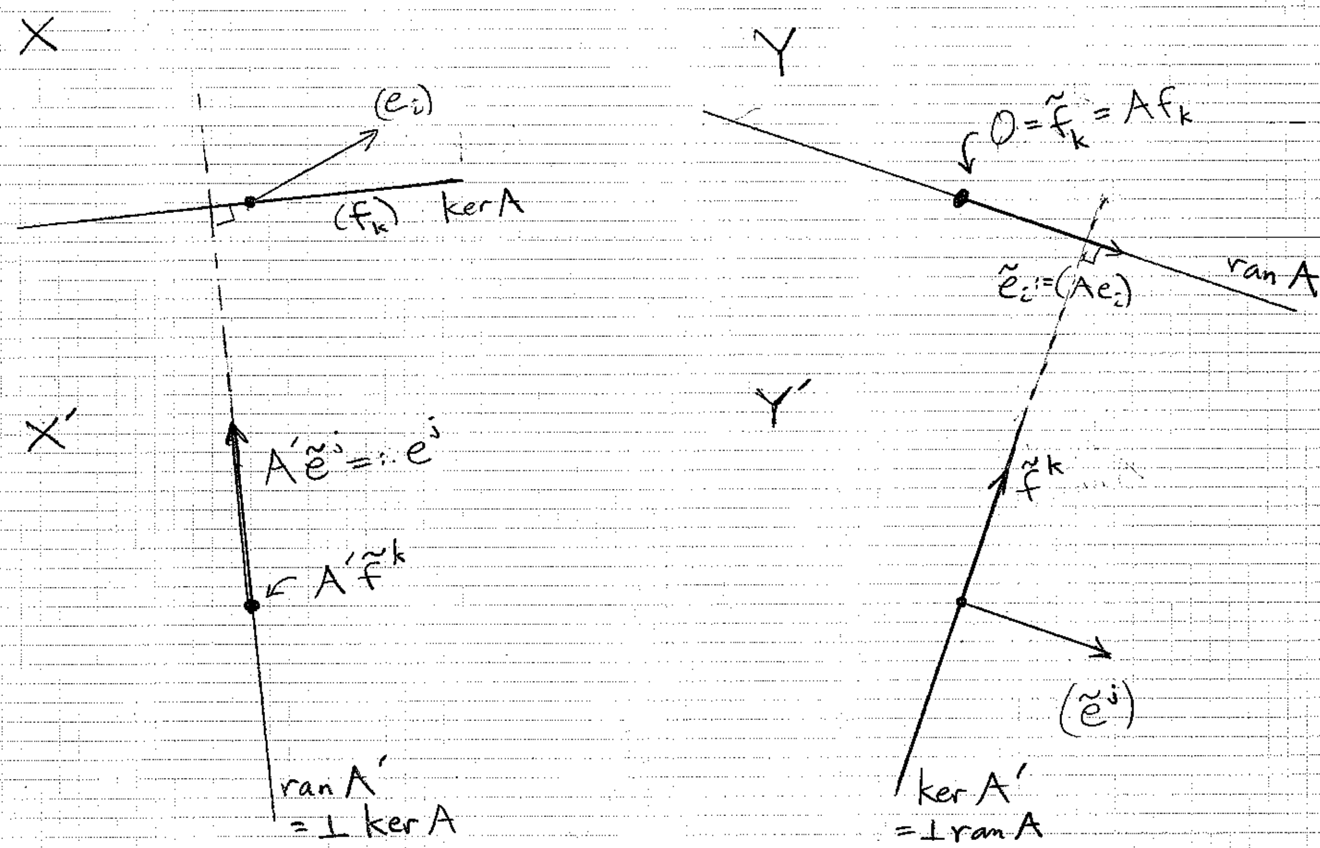
# How to picture the adjoint of a noninvertible map



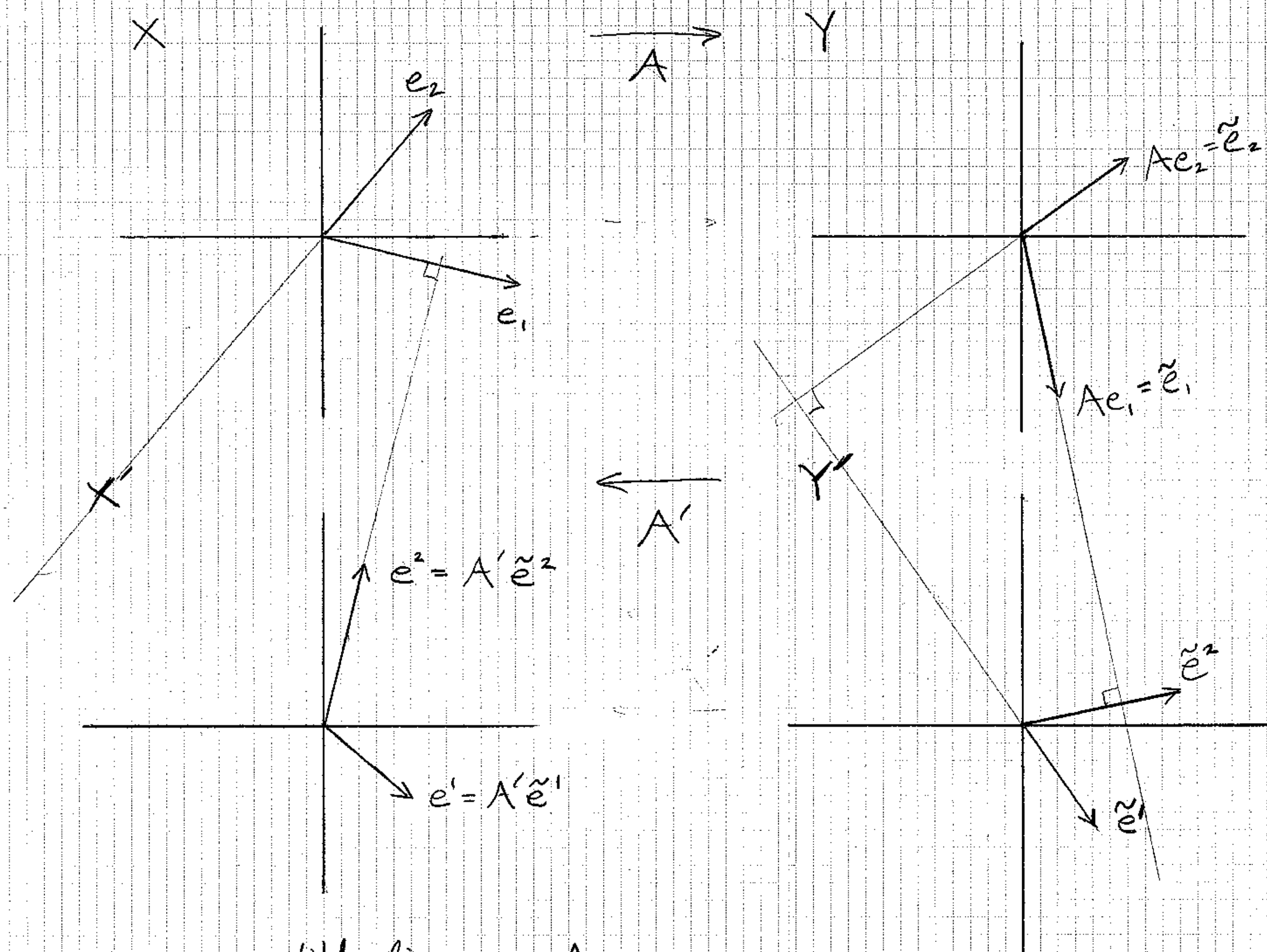

This reduces to the invertible case if we:

- mod out $X$ by $\ker A$
- restrict $X'$ to $\operatorname{ran} A' = \perp \ker A$
- restrict $Y$ to $\operatorname{ran} A$
- mod out $Y'$ by $\ker A' = \perp \operatorname{ran} A$

So we:

- represent $X$ by a Hamel basis which extends a Hamel basis for $\ker A$
- represent $Y'$ by a Hamel basis which:
  - extends a Hamel basis for $\ker A' = \perp \operatorname{ran} A$
  - includes the dual set of $(\tilde{e}_i)$
  - is further extended to give a Hamel basis for all of $Y'$ in case $\dim Y' < \infty$.

# How to picture the adjoint of an invertible map




- For an invertible linear map $A$, $A'$ maps the dual basis of ran $A$ to the dual basis of dom $A$.

  Specifically:

  Let $\tilde{e}^j A e_i = \delta^j_i$

  $\Leftrightarrow \underbrace{(A'\tilde{e}^j)}_{e^j} e_i = \delta^j_i$

- If $\dim X = \dim Y \not< \infty$,

  To fully specify $A'$

  extend $(\tilde{e}^j)$ to a Hamel basis $(\tilde{e}^j) \cup (\tilde{h}^k)$

  $A'\tilde{h}^k$ is fully specified by what it does to $(e_i)$,

  which is identical to what $\tilde{h}^k$ does to $(Ae_i)$